Extra Homework Assignments

Bifurcations: Theory and Applications

Bernold Fiedler, Alejandro López Nieto

http://dynamics.mi.fu-berlin.de/lectures/

due date: Wednesday, December 4, 2019, 12:00

In the following W and Z are Banach spaces with norms denoted by $||\cdot||_W$ and $||\cdot||_Z$, respectively. $\mathcal{L}(W,Z)$ denotes the space of bounded linear operators from W to Z.

Theorem: [Open mapping theorem]
Let $L \in \mathcal{L}(W,Z)$ be surjective. Then L is open, i.e. L maps open sets in W to open sets in Z.

Theorem: [Bounded inverse theorem]
Let $L \in \mathcal{L}(W,Z)$ be bijective. Then $L^{-1} \in \mathcal{L}(Z,W)$.

Theorem: [Closed graph theorem]
Let $L : W \to Z$ (not assumed to be bounded!) possess a closed graph, i.e. $\mathrm{gr}(L) \coloneqq \{(w, L(w)) \mid w \in W\}$ is closed in $(W \times Z, ||\cdot|| \coloneqq ||\cdot||_W + ||\cdot||_Z)$. Then L is bounded.

Problem 1:
Show that the open mapping theorem implies the bounded inverse theorem.

Problem 2:
Show that the bounded inverse theorem implies the closed graph theorem.

Problem 3:
Show that the closed graph theorem implies the bounded inverse theorem.

Problem 4: Show that $W = U \oplus V$ for closed linear subspaces U, V, if, and only if, there exists a bounded linear projection $P : W \to U$ along V, i.e. a bounded linear operator P such that $P^2 = P$, range $P = U$ and ker $P = V$.
Let $Q \coloneqq I - P$ for P satisfying the conditions above. Show that Q is a bounded linear projection onto V along U.