Homework Assignments

Dynamical Systems I

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http://dynamics.mi.fu-berlin.de/lectures/

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Form teams of two, work on four problems per week, submit at least two, get one right each week (on average). Submission procedure: email a .pdf or .jpg file of your solutions to your assigned tutor, before the deadline. Be prepared to explain any of your solutions (no matter whether your own or the solution by your team partner!) during any tutorial, live, on webex.

Problem 1: Consider a flow $\Phi(t,x) = \Phi_t(x)$ on the real axis, $x \in \mathbb{R}$.

(i) Prove or disprove: any periodic orbit is stationary, i.e.

$$\exists\, p > 0 \ : \ \Phi(p, x_0) = x_0 \quad \Longrightarrow \quad \forall\, t \in \mathbb{R} \ : \ \Phi(t, x_0) = x_0$$

(ii) What are possible α-limit and ω-limit sets of an orbit of Φ? (Consider bounded and unbounded orbits.)

Problem 2: Consider a flow Φ on $\mathbb{R}^N$, $N \geq 2$. The orbit $\{\Phi_t(x_0) \,|\, t \in \mathbb{R}\}$ of x_0 is assumed to possess period p', i.e.,

$$\forall\, t \in \mathbb{R} : \Phi_{t+p'}(x_0) = \Phi_t(x_0).$$

The smallest positive number p' which satisfies the above (if it exists) is called the *minimal period* p.

In addition, assume that the orbit $\Phi_t(x_0)$ of x_0 is nonstationary, i.e.,

$$\exists\, t^* \in \mathbb{R} : \Phi_{t^*}(x_0) \neq x_0.$$

Prove or disprove: The orbit $\Phi_t(x_0)$ possesses a minimal period $p > 0$.

Problem 3: Consider the map

$$\Phi_t(x) := \begin{cases} \dfrac{1}{\frac{1}{x} - t} & \text{for } x \neq 0 \text{ and } \frac{1}{x} \neq t \\ 0 & \text{for } x = 0 \end{cases} \tag{1}$$

for $x \in \mathbb{R}$ and $t \in \mathbb{R} \setminus \{1/x\}$.

(i) Check the local flow properties for Φ_t and determine the minimal time $\underline{t}(x_0)$ and the maximal time $\overline{t}(x_0)$ of existence for every $x_0 \in \mathbb{R}$.

(ii) Which vector field is associated to Φ_t?

Problem 4: Consider again the map (1) from problem 3, but now with $x \in \mathbb{C}$.

(i) Determine the α- and ω-limit sets $\alpha(x_0)$ and $\omega(x_0)$ for every $x_0 \in \mathbb{C}$.

(ii) Find all bounded / unbounded / stationary / periodic / homoclinic / heteroclinic trajectories.