Homework Assignments

Dynamical Systems I

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`http://dynamics.mi.fu-berlin.de/lectures/`

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Problem 5: Consider the vector field $f : \mathbb{R}^2 \to \mathbb{R}^2$ defined by

$$\dot{x} \;=\; \begin{pmatrix} a & -b \\ b & a \end{pmatrix} x,$$

with $a, b \in \mathbb{R}$. Transform this linear differential equation into polar coordinates:

$$x \;=\; \begin{pmatrix} r\cos\phi \\ r\sin\phi \end{pmatrix},$$

with $r > 0$, $\phi \in \mathbb{R}/2\pi\mathbb{Z}$. Choose $b \neq 0$ arbitrarily and sketch phase portraits in (r, ϕ)-coordinates and in x-coordinates for $a < 0$, $a = 0$, $a > 0$.

Problem 6: Let $\Phi_{t,s} : \mathbb{R}^N \to \mathbb{R}^N$ be a *periodic* evolution with period 1, i.e.,

$$\text{for all } t, s \in \mathbb{R} : \quad \Phi_{t+1,s+1} = \Phi_{t,s}.$$

Consider the *stroboscope* map $\Pi : \mathbb{R}^N \to \mathbb{R}^N$,

$$\Pi(x) = \Phi_{1,0}(x).$$

Prove:

(i) for all $t, s \in \mathbb{R}$, the following holds:

$$\Phi_{t,s} = \Phi_{t-[t],0} \circ \Pi^{[t]-[s]} \circ \left(\Phi_{s-[s],0}\right)^{-1};$$

(ii) for each $t \in \mathbb{R}$ there exists a change of coordinates $\Psi : \mathbb{R}^N \to \mathbb{R}^N$ such that for all $k \in \mathbb{N}$: $\Phi_{t+k,t} = \Psi^{-1}\Pi^k\Psi$. Determine Ψ.

Problem 7: Identify the ω-limits of <u>each</u> trajectory of

(i) $\dot{x} = \sin(x), \qquad x \in \mathbb{R},$

(ii) $\begin{pmatrix} \dot{x_1} \\ \dot{x_2} \end{pmatrix} = \begin{pmatrix} \cos 2x_2 \\ \cos x_1 \end{pmatrix}, \qquad x = (x_1, x_2) \in \mathbb{R}^2.$

Problem 8: Consider an equilibrium x_0 of a flow φ_t in $X = \mathbb{R}^n$ and neighborhoods $U \subset V$ of x_0 in X such that each forward trajectory $\gamma(x) := \{\varphi_t(x), t \geq 0\}$ of a point $x \in U$ stays in V and converges to x_0 as $t \to \infty$.

Prove or disprove: Every first integral of φ is constant in U.