Homework Assignments

Dynamical Systems I

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http://dynamics.mi.fu-berlin.de/lectures/

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Problem 9: In a windowless two-dimensional lab with a given force field $f(x)$, a physics student tries to check on Newton's Law: force = mass times acceleration, i.e. $f(x) = m\ddot{x}$. She is working with force free particles, i.e., $f(x) = 0$. Unfortunately, an evil and invisible demon has abducted her and keeps rotating the locked lab at constant angular velocity ω, so that the student can only measure

$$y(t) := R(-\omega t)x(t),$$

where $R(\alpha)$ is the rotation in the plane by angle α.

Show how the smart student discovers two forces acting on y:

(i) the *centrifugal force* in the direction of x, and

(ii) the *Coriolis force* orthogonal to the velocity $\dot{x}$.

Derive the formulas for both forces, in the plane, and help her to discover the rotation speed ω.

Free extra points: Can she discover and determine the rotation of Earth, from her windowless lab? Assume that the lab is located at the north pole. It is sufficient to consider a linear pendulum. Hint: her name is Leonie Foucault.

Problem 10: A coughing (point-sized) student chases his/her (point-sized) professor. The professor starts from his office at the origin $(x, y) = (0, 0)$ of the plane $\mathbb{R}^2$ and runs along the positive x-axis with speed 1. At the same moment a (point-sized) student —with loads of tricky questions concerning the homework assignment— starts at the point $(x, y) = (0, d)$ and chases the professor. The student has the same speed 1 and always runs directly towards the professor.

Which initial distance d guarantees that the professor can keep a social distancing of 1.5 m?

Hint: Use appropriate coordinates (e.g. r = distance of student and professor, φ = angle of the x-axis with the connecting line of both persons) and solve the resulting system by separation of variables.

Problem 11: Consider the two vector fields

(i) $\dot{x} \;=\; f(x) \;:=\; (1+|x|^2)x \;\in\; \mathbb{R}^2.$

(ii) $\dot{y} \;=\; g(y) \;:=\; y \;\in\; \mathbb{R}^2,$

Prove or disprove that f-orbits and g-orbits coincide.

Warning: The theorem stated in the lecture is not applicable because the flow of (i) does not exist globally.

Problem 12: Consider a radially symmetric vector field in the plane,

$$\begin{aligned} \dot{x} &= -x - g(x^2+y^2)y, \\ \dot{y} &= g(x^2+y^2)x - y. \end{aligned} \tag{1}$$

(i) Find an Euler multiplier $\mu = \mu(x^2+y^2)$ that turns it into a divergence-free vector field.

(ii) Find an example for a function $g(x^2+y^2)$ such that the system (1) does *not* possess a nontrivial First Integral.

(iii) What is wrong?