

## Homework Assignments

### Dynamical Systems I

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<http://dynamics.mi.fu-berlin.de/lectures/>

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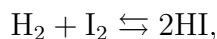
**Problem 21:** Use your favorite numerical integrator to plot the solution of the Van-der-Pol oscillator

$$\begin{aligned}\varepsilon \dot{x} &= -y + x(1 - x^2) \\ \dot{y} &= x\end{aligned}$$

with initial conditions  $x(0) = 1$ ,  $y(0) = 0$  up to time  $t = 10$ . Choose parameters  $\varepsilon = 0.16, 0.01, 0.0025, 0.0009$ , and  $0.000625$ . Use stepsize  $h = 10^{-3}$ . Compare explicit Euler, Runge Kutta, and another solver of your choice. Describe and discuss observations and problems.

*Extra credit:* What happens for  $\varepsilon = 0.0009$  to the numerical solution calculated by the Runge-Kutta solver with fixed stepsize  $h = 10^{-3}$ ?

**Problem 22:** Consider the synthesis of hydrogen iodide from hydrogen and iodine,



is well described by mass-action kinetics. Denote the reaction rates by  $k_{\rightarrow}$ ,  $k_{\leftarrow}$  and determine the kinetic equations for the vector  $x = (x_{\text{H}_2}, x_{\text{I}_2}, x_{\text{HI}})$  of concentrations. Determine the  $\omega$ -limit to initial data  $x^0 = (x_{\text{H}_2}^0, x_{\text{I}_2}^0, 0)$ .

*Extra credit:* Which choice of initial data yields the highest gain?

**Problem 23:** Consider the initial-value problem

$$\dot{x} = f(x) := Ax, \quad x(0) = x_0,$$

$A \in \mathbb{R}^{n \times n}$  on the time interval  $0 \leq t \leq T$  for fixed  $T > 0$ . Calculate an approximate solution  $x(T)$  analytically by Picard iteration, i.e., determine the  $n$ -th Picard iterate  $x^{[n]}(T)$ ;

$$x^{[k+1]}(t) = x_0 + \int_0^t f(x^{[k]}(\tau)) \, d\tau, \quad x^{[0]}(t) \equiv x_0.$$

Now consider  $A = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$  and  $x_0 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ .

How fast do the Picard iterations approximate the exact solution?

**Problem 24:** The initial-value problem

$$\dot{x} = f(x) := x^2, \quad x(0) = x_0 := 2$$

has a solution for  $-\infty < t < \bar{\tau}$  with “blow-up”,  $\lim_{t \rightarrow \bar{\tau}} x(t) = +\infty$ .

Let  $(x_k)_{k \in \mathbb{N}}$  be the series of Picard iterates:

$$\begin{aligned} x_0(t) &\equiv x_0, \\ x_{n+1}(t) &= x_0 + \int_0^t f(x_n(s)) \, ds. \end{aligned}$$

- (i) Prove:  $x_k(t)$  is defined for all  $k \in \mathbb{N}$  and  $t \in \mathbb{R}$ .
- (ii) Calculate  $x_1(t), x_2(t), x_3(t)$  and  $x_4(t)$  explicitly.
- (iii) Determine all  $t \geq 0$  such that  $x_k(t)$  converges to the solution  $x(t)$  of the initial-value problem, as  $k \rightarrow \infty$ .