Homework Assignments

Dynamical Systems I

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http://dynamics.mi.fu-berlin.de/lectures/

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Problem 21: Use your favorite numerical integrator to plot the solution of the Van-der-Pol oscillator

$$\begin{aligned} \varepsilon\dot{x} &= -y + x(1-x^2) \\ \dot{y} &= x \end{aligned}$$

with initial conditions $x(0) = 1$, $y(0) = 0$ up to time $t = 10$. Choose parameters $\varepsilon = 0.16$, 0.01, 0.0025, 0.0009, and 0.000625. Use stepsize $h = 10^{-3}$. Compare explicit Euler, Runge Kutta, and another solver of your choice. Describe and discuss observations and problems.

Extra credit: What happens for $\varepsilon = 0.0009$ to the numerical solution calculated by the Runge-Kutta solver with fixed stepsize $h = 10^{-3}$?

Problem 22: Consider the synthesis of hydrogen iodide from hydrogen and iodine,

$$H_2 + I_2 \leftrightarrows 2HI,$$

is well described by mass-action kinetics. Denote the reaction rates by $k_\rightarrow$, $k_\leftarrow$ and determine the kinetic equations for the vector $x = (x_{H_2}, x_{I_2}, x_{HI})$ of concentrations. Determine the ω-limit to initial data $x^0 = (x^0_{H_2}, x^0_{I_2}, 0)$.

Extra credit: Which choice of initial data yields the highest gain?

Problem 23: Consider the initial-value problem

$$\dot{x} = f(x) := Ax, \qquad x(0) = x_0,$$

$A \in \mathbb{R}^{n\times n}$ on the time interval $0 \le t \le T$ for fixed $T > 0$. Calculate an approximate solution $x(T)$ analytically by Picard iteration, i.e., determine the n-th Picard iterate $x^{[n]}(T)$;

$$x^{[k+1]}(t) = x_0 + \int_0^t f(x^{[k]}(\tau))\,\mathrm{d}\tau, \qquad x^{[0]}(t) \equiv x_0.$$

Now consider $A = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ and $x_0 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$.

How fast do the Picard iterations approximate the exact solution?

Problem 24: The initial-value problem

$$\dot{x} = f(x) := x^2, \qquad x(0) = x_0 := 2$$

has a solution for $-\infty < t < \bar{\tau}$ with "blow-up", $\lim_{t\to\bar{\tau}} x(t) = +\infty$.

Let $(x_k)_{k\in\mathbb{N}}$ be the series of Picard iterates:

$$\begin{aligned} x_0(t) &\equiv x_0, \\ x_{n+1}(t) &= x_0 + \int_0^t f(x_n(s))\,\mathrm{d}s. \end{aligned}$$

(i) Prove: $x_k(t)$ is defined for all $k \in \mathbb{N}$ and $t \in \mathbb{R}$.

(ii) Calculate $x_1(t), x_2(t), x_3(t)$ and $x_4(t)$ explicitly.

(iii) Determine all $t \geq 0$ such that $x_k(t)$ converges to the solution $x(t)$ of the initial-value problem, as $k \to \infty$.