Homework Assignments

Dynamical Systems I

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http://dynamics.mi.fu-berlin.de/lectures/

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Problem 25: Show that

$$w(t) = \exp\left(\int_{t_0}^{t} \beta(\sigma)\,\mathrm{d}\sigma\right)\left(w_0 + \int_{t_0}^{t} \exp\left(-\int_{t_0}^{s} \beta(\sigma)\,\mathrm{d}\sigma\right)\alpha(s)\,\mathrm{d}s\right)$$

is the solution of the initial-value problem

$$\dot{w}(t) = \alpha(t) + \beta(t)w(t), \qquad w(t_0) = w_0.$$

Problem 26: Let $f : X \to X = \mathbb{R}^n$ be locally Lipschitz continuous and $J(x_0) = (t_-(x_0), t_+(x_0))$ the maximal interval of existence of the solution to the initial-value problem

$$\dot{x}(t) = f(x(t)), \qquad x(0) = x_0.$$

Is the map $x_0 \mapsto t_+(x_0) \in (0, \infty]$ continuous?

Problem 27: Consider a continuously differentiable vector field $f : X \times \mathbb{R} \to X = \mathbb{R}^n$. Let $x(t, t_0)$ denote the solution at time t of the associated initial-value problem

$$\dot{x}(t) = f(x(t), t), \qquad x(t_0) = x_0.$$

Prove: For any fixed τ such that $x(\tau, t_0)$ exists, there exists a neighborhood U of t_0 such that the map

$$(t_0 - \varepsilon, t_0 + \varepsilon) \to X, \qquad s \mapsto x(\tau, s) = \Phi_{\tau,s} x_0,$$

is differentiable with respect to s, for $s \in U$. Which differential equation is solved by $v(t) := \mathrm{D}_{t_0} x(t, t_0)$?

Problem 28: Consider $\dot{x} = f(x)$, $f \in C^1$, $f(0) = 0$, $x \in \mathbb{R}^N$ with flow Φ_t.

Prove or disprove: The linearization of a flow, at an equilibrium $x_0 = 0$, is the flow of the linearized vector field, at $x_0 = 0$, i.e., $(\Phi_t)'(0) = \exp(f'(0)t)$.