Homework Assignments

Dynamical Systems I

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Problem 37: Consider the forced damped linear pendulum

$$\ddot{x} + \nu \dot{x} + \omega_0^2 x = f \cos(\omega t)$$

with parameters $f, \nu, \omega_0, \omega > 0$ and initial conditions $x(0) = 0$, $\dot{x}(0) = 1$.

(i) Does there exist a unique periodic solution?

(ii) Calculate its amplitude $A(\omega)$ and sketch it.

(iii) For which parameter combinations does there exist a unique global maximum $A(\omega_{max})$? Show that $\omega_{max} < \omega_0$.

Hint: We call ω_{max} the *resonant frequency* .

Problem 38: How many digits (in the decimal system) does the 202.020.202.020-th entry of the sequence $(1, 3, 8, 20, 48, 112, \ldots)$ have, i.e. $x_n = 4x_{n-1} - 4x_{n-2}$ with $x_0 = 1$ and $x_1 = 3$?

Problem 39: Consider the system of differential equations

$$\dot{x}_i \;=\; x_i \left((Ax)_i - x^{\mathrm{T}} Ax \right), \qquad i = 1, \ldots, N,$$

for $x = (x_1, \ldots, x_N) \in \mathbb{R}^N$ with $x_i \geq 0$ and

$$\sum_{i=1}^{N} x_i \;=\; 1.$$

Consider the case of the identity matrix, $A = \mathrm{id}$.

(i) Sketch the phase portraits for $N = 2, 3, 4$.

(ii) Describe the set of equilibria and the set of heteroclinic orbits for arbitrary N. In particular determine which equilibria are connected by heteroclinic orbits.

Extra credit: What happens for $\sum_{i=1}^{N} x_i \neq 1$ and/or $x_i < 0$ for some i ?

Hint: Enumerate equilibria x^* by subsets $M(x^*) = \{i | x_i^* \neq 0\} \subseteq \{1, \ldots, N\}$.

Problem 40: Let f be a differentiable vector field such that each trajectory is bounded.

Prove or disprove: The ω-limit depends continuously on the initial condition, i.e. if

$$\lim_{n\to\infty} \operatorname{dist}(x_n, x) \;=\; 0,$$

then

$$\lim_{n\to\infty} \operatorname{dist}(\omega(x_n), \omega(x)) \;=\; 0.$$

Here we use the symmetric Hausdorff distance

$$\operatorname{dist}(A, B) \;:=\; \max\left(\sup_{a\in A}\inf_{b\in B} \operatorname{dist}(a, b)\,,\; \sup_{b\in B}\inf_{a\in A} \operatorname{dist}(a, b)\right).$$