Homework Assignments

Dynamical Systems I

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http://dynamics.mi.fu-berlin.de/lectures/

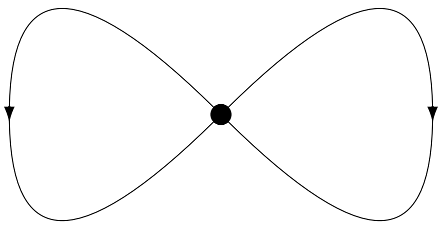
due date: Thursday, July 9, 2020, 23:59

Problem 41: Let f be a differentiable vector field on $\mathbb{R}^3$.

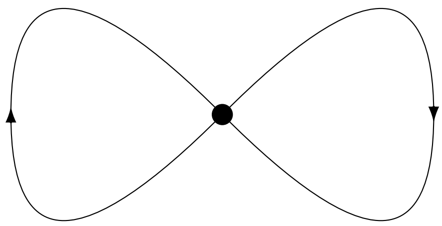
Prove or disprove: A trajectory coincides with its ω-limit set if, and only if, the trajectory is an equilibrium or a periodic orbit.

Problem 42: Which of the following sets are possible ω-limits of a (single) trajectory of some planar flow? Which of the sets cannot occur as ω-limits (of a single trajectory)? Justify your claims, without providing explicit vector fields.

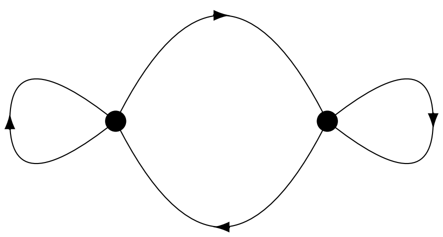
(i)



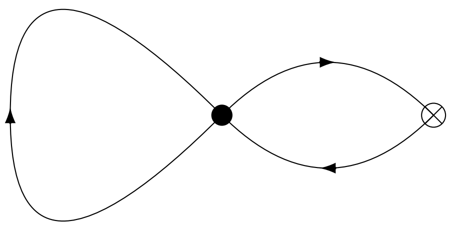
(ii)



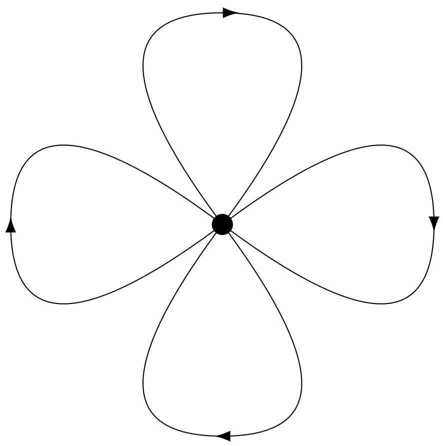
(iii)



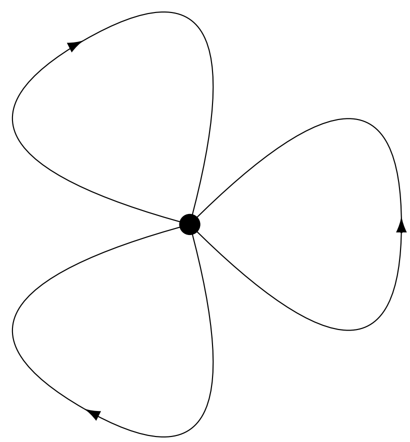
(iv)



(v)



(vi)



Discs ● denote equilibria (of any type) and crossed out circles ⊗ denote hyperbolic saddles.

Problem 43: Let $A \subseteq B \subseteq X = \mathbb{R}^N$ be sets and φ_t a flow on X. The set A is called *chain-recurrent* with respect to B if for every $y_0 \in A$ and every $\varepsilon > 0$, $T > 0$ there exists a positive number $n \in \mathbb{N}$, a sequence of times $t_0, \ldots, t_{n-1} \geq T$, and points $y_1, \ldots, y_{n-1} \in B$ such that

$$\operatorname{dist}\left(\varphi_{t_i}(y_i), y_{i+1}\right) < \varepsilon, \qquad i = 0, \ldots, n-1 \quad (\text{mod } n, \text{ i.e. } y_n := y_0).$$

The set A is called *recurrent*, if we can chose chains of length $n = 1$ for all points, i.e. if $y_0 \in \omega(y_0)$ for all $y_0 \in A$.

Prove: For any $x_0 \in X$, the ω-limit $\omega(x_0)$ is chain-recurrent with respect to X, but it is not necessarily recurrent.

Extra credit: Let the trajectory $\varphi_t(x_0)$ be bounded. Prove or disprove: The ω-limit $\omega(x_0)$ is chain-recurrent with respect to *itself*.

Problem 44: Prove or disprove the theorem of POINCARÉ & BENDIXSON for flows on

(i) the sphere S^2,

(ii) the three-dimensional torus T^3.