Dynamical Systems 1: basic questions

31. When do we call a set $M$ positively invariant, negatively invariant, or invariant with respect to a given flow $\Phi_t$?

32. Let the forward orbit $\gamma^+(x_0)$ of $x_0$ under a flow in $\mathbb{R}^N$ be bounded. Is the $\omega$-limit set of $x_0$

(a) open,
(b) closed,
(c) bounded,
(d) compact,
(e) stable,
(f) asymptotically stable,
(g) unstable?

Why?

33. Let the forward orbit $\gamma^+(x_0)$ of $x_0$ under a flow in $\mathbb{R}^N$ be bounded. Is the $\omega$-limit set of $x_0$

(a) nonempty,
(b) finite,
(c) discrete,
(d) connected,
(e) positively invariant,
(f) negatively invariant,
(g) invariant?

Why?

34. How is a Lyapunov function of a vector field defined? When do we call a Lyapunov function strict?

35. How does a Lyapunov function restrict possible $\omega$-limit sets of a flow?

36. Formulate the invariance principle of LaSalle.

37. How are stability and asymptotic stability of a nonempty, compact, invariant set defined?

38. Why can the Brusselator

$$\dot{x}_1 = a - x_1 - bx_1 + x_1^2 x_2, \quad \dot{x}_2 = bx_1 - x_1^2 x_2, \quad a, b > 0,$$

possess a nonstationary periodic orbit? Can it possess a *stable* nonstationary periodic orbit?
39. Formulate the theorem of Poincaré & Bendixson on $\omega$-limit sets of bounded orbits of planar vector fields.