Homework Assignments

Dynamical Systems II

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http://dynamics.mi.fu-berlin.de/lectures/

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Problem 1: Let x_* be an equilibrium of

$$\dot{x} = f(x).$$

Consider the linearized equation at x_*

$$\dot{y} = f'(x_*)y.$$

Assume $x = x_*$ is not stable. Can the equilibrium $y = 0$ of the linearized equation be stable? Can it be asymptotically stable?
Consider now that $x = x_*$ is asymptotically stable. Can the equilibrium $y = 0$ of the linearized equation fail to be stable?

Problem 2: Consider the real iteration

$$x_{n+1} = f_a(x_n) := ax_n(1 - x_n^2),$$

with parameter $a > 0$. Determine the fixed points $x_* = f_a(x_*)$ and discuss their linearized stability for $0 < a < 1$, $1 < a < 2$ and $a > 2$.

Problem 3: Consider matrices B_i, $i = 0, 1$, with all eigenvalues in the interior of the complex unit disk.Is the fixed point $x = 0$ of the alternating iteration

$$x_{n+1} = \begin{cases} B_0 x_n, & n \text{ even}, \\ B_1 x_n, & n \text{ odd}, \end{cases}$$

always stable?

Problem 4: Let $I \subset \mathbb{R}$ be an interval and $A \in C^1(I, \mathbb{R}^{N\times N})$.

Prove: If A and $\dot{A}$ commute, i.e. if $[A(t), \dot{A}(t)] := A(t)\dot{A}(t) - \dot{A}(t)A(t) = 0$ for all $t \in I$, then

$$\frac{\mathrm{d}}{\mathrm{d}t} e^{A(t)} = \dot{A}(t)\, e^{A(t)} = e^{A(t)}\, \dot{A}(t).$$