Homework Assignments

Dynamical Systems II

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http://dynamics.mi.fu-berlin.de/lectures/

due date: Thursday, November 26, 2020, 12:00

Problem 5: Consider real-valued solutions $u = u(t,x)$, $t,x \in \mathbb{R}$, of the partial differential equation

$$\tfrac{\partial}{\partial t} u = \tfrac{\partial^2}{\partial x^2} u + f(u),$$

A traveling wave with wave speed $c \in \mathbb{R}$ is a solution of the special form $u(t,x) = U(x - ct)$. Derive an ordinary differential equation that is satisfied by U.

Now consider the cubic nonlinearity $f(u) := u(1-u)(u-a)$, for any fixed parameter $0 < a < 1$. Show that $U(\xi) := 1/(1 + \exp(-k\xi))$ is a traveling wave, for suitable k. Determine the wave speed c of the traveling wave U.

Problem 6: Floquet theory studies the Wronskian $W(t, t_0)$ of non-autonomous linear systems $\dot{y}(t) = A(t)y(t)$, with matrices $A(t+p) = A(t)$ of period p in t.

(i) Show $W(np, 0) = W(p, 0)^n$, for all $n \in \mathbb{N}$.

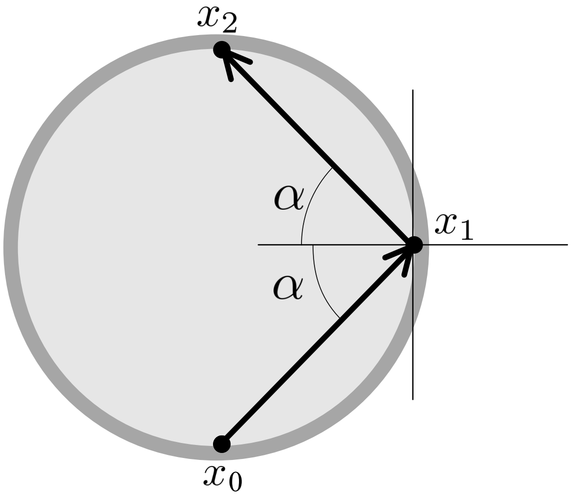
(ii) Compare the spectra of $W(p,0)$, $W(p+t_0, t_0)$, $W(np, 0)$ and $W(np+t_0, t_0)$.

Problem 7: Consider a homeomorphism $A : \mathbb{R} \to \mathbb{R}$ satisfying

$$A(x + 2\pi) = A(x) - 2\pi,$$

for all $x \in \mathbb{R}$.
Prove or disprove: the induced map $\tilde{A} := A(\text{mod } 2\pi)$ from S^1 to $S^1 := \mathbb{R}/2\pi\mathbb{Z}$ possesses a fixed point.

Problem 8: An ideal point-sized billiard ball moves with constant speed in the unit disc $B^2 \subset \mathbb{R}^2$. Fix an initial position x_0 on the boundary and a shooting angle α. Depending on α, describe the ω-limit set of the sequence of points $\{x_n\}_{n\in\mathbb{N}}$ in S^1, where the ball hits the boundary.



The ω-limit set of $\{x_n\}_{n\in\mathbb{N}}$ is defined as

$$\omega(x_0) \coloneqq \{y \in S^1 \mid \text{there exists a subsequence } n_k \xrightarrow{k\to\infty} \infty \text{ for which } x_{n_k} \to y\}$$