Homework Assignments

Dynamical Systems II

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http://dynamics.mi.fu-berlin.de/lectures/

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Problem 13: Consider the ordinary differential equation on the circle

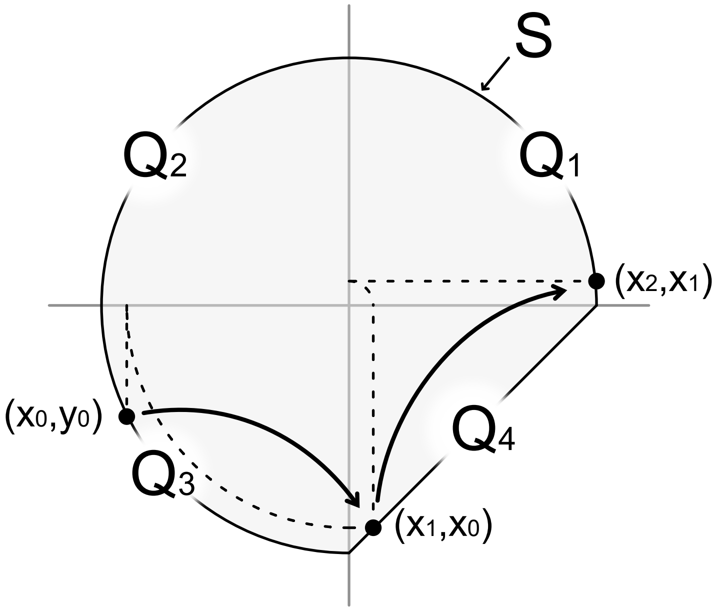
$$\dot{x} = f(x), \qquad x \in S^1, \quad f(x) > 0$$

with corresponding flow Φ_t. Determine a formula for the rotation number $\varrho(\Phi_T)$ of the time-T map. Does a devil's staircase in $\varrho(\Phi_T)$ arise with respect to the parameter T?

Problem 14: Consider a nonlinear C^2-flow Φ_t on $\mathbb{T}^2 = \mathbb{R}^2/2\pi\mathbb{Z}^2$ with the y-axis S^1 as global Poincaré section. Let the first return time be identically 2π and assume that the rotation number ϱ of $\Phi_{2\pi}$ is irrational. Show that there exists a homeomorphism h of $\mathbb{T}^2$ which conjugates Φ_t to the parallel flow generated by

$$\begin{aligned} \dot{x} &= 1, \\ \dot{y} &= \varrho. \end{aligned}$$

Problem 15: Let A be the circle map described by the following diagram:



Here Q_i, i (mod 4), denote the quadrants of the depicted cut circle S, with a straight lower right secant. A maps any point (x_0, y_0) in quadrant Q_i to the unique point of S in quadrant Q_{i+1} whose y-component coincides with x_0.

What is the rotation number ϱ of A? Does there exist a homeomorphism h conjugating A to a rigid rotation?

Problem 16: Consider the Fibonacci iteration on the torus

$$\begin{aligned} A : \mathbb{T}^2 &\to \mathbb{T}^2 = \mathbb{R}^2/\mathbb{Z}^2, \\ (x, y) &\mapsto (y, x + y) (\text{mod } 1). \end{aligned}$$

Is A well-defined? Calculate stable and unstable manifolds of the fixed point $(0, 0)$ under the iteration A^n. Are they dense on the torus?