Homework Assignments

Dynamical Systems II

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http://dynamics.mi.fu-berlin.de/lectures/

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Problem 21: Let $\mathrm{P}(n)$ be the number of periodic points of a map Φ with (not necessarily minimal) period n. A measure of the complexity of Φ is given by the *topological entropy* h defined as

$$h := \limsup_{n\to\infty} \frac{\log \mathrm{P}(n)}{n}.$$

Calculate the entropy h of the shift on N symbols.
Prove that every iteration Φ containing shift dynamics (i.e. with an invariant set I such that $\Phi|_I$ is conjugate to a shift on m symbols) has positive topological entropy.

Problem 22: Let Φ be a homeomorphism containing shift dynamics with invariant set I. We denote by $\tau : S \to I$ a homeomorphism which conjugates $\Phi|_I$ to the shift σ on S with symbols $\{1, \ldots, N\}$. Consider $\tau_\pi = \tau \circ H_\pi$, where $H_\pi : S \to S$ is the trivial homeomorphism induced by a permutation $\pi : \{1, \ldots, N\} \to \{1, \ldots, N\}$. In other words,

$$H_\pi(\mathbf{s})_k = \pi(\mathbf{s}_k).$$

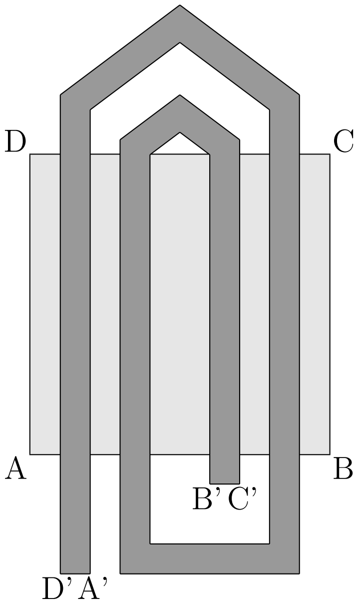
Prove or disprove:

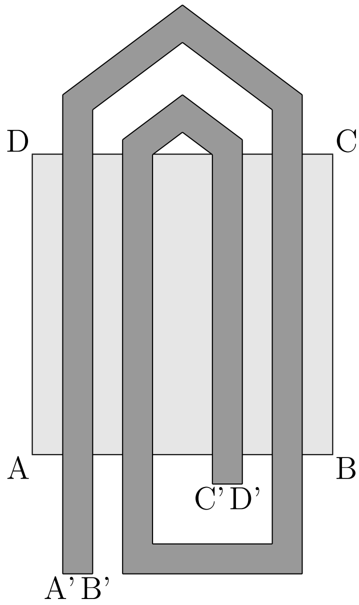
(i) The homeomorphism τ_π conjugates $\Phi|_I$ to the shift σ.

(ii) Let $\rho : S \to I$ be a homeomorphism which conjugates $\Phi|_I$ to the shift σ, then there exists a permutation $\pi : \{1, \ldots, N\} \to \{1, \ldots, N\}$ such that

$$\rho = \tau_\pi.$$

Problem 23: Which of the following "paper-clip" maps contains shift dynamics? (Assume the maps to be affine linear, in the regions of intersection.)





Problem 24: Calculate all fixed points of the bouncing-ball map f:

$$\begin{aligned} \Phi_{j+1} &= \Phi_j + v_j, \\ v_{j+1} &= \alpha\, v_j - \gamma \cos(\Phi_j + v_j), \end{aligned}$$

with $\Phi_j \in S^1 = \mathbb{R}/(2\pi\mathbb{Z})$ and $v_j \in \mathbb{R}$, for $0 < \alpha < 1$ and $0 < \gamma$. How many fixed points does f have, for given α, γ? Determine the type (hyperbolically stable, hyperbolically unstable, non-hyperbolic) of the fixed points. Sketch the dependence of the fixed points on γ, for $\alpha = \frac{1}{2}$. What happens for $\alpha \to 1$?