

Homework Assignments

Dynamical Systems II

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<http://dynamics.mi.fu-berlin.de/lectures/>

due date: Thursday, February 4, 2020, 16:00

Problem 29: Consider a diffeomorphism $\Phi : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ with a transverse homoclinic orbit to a hyperbolic fixed point at 0. In class, we proved that an iterate Φ^n contains shift dynamics on 2 symbols.

Prove that for every $N \in \mathbb{N}$ there exists some iterate Φ^n containing shift dynamics on N symbols.

Problem 30: Consider a diffeomorphism $\Phi : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ having a hyperbolic structure on the compact, invariant subset I . In other words, there exist $0 < \lambda < 1$ and a continuous invariant splitting $\mathbb{R}^2 = X_p \oplus Y_p$ for all $p \in I$ such that $|D\Phi(p)x| \leq \lambda|x|$ and $|D\Phi(p)y| \geq |y|/\lambda$ for all $(x, y) \in X_p \times Y_p$.

Prove or disprove: the hyperbolic structure on I is unique.

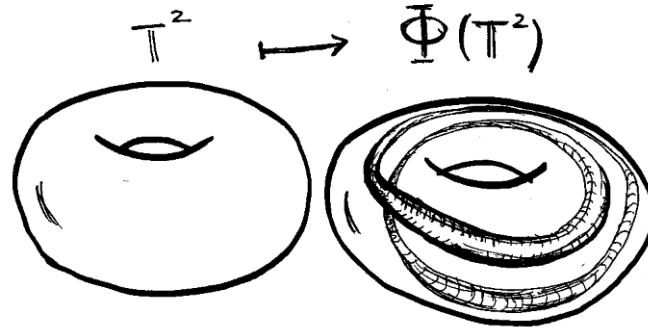
Problem 31: Consider a C^2 differential equation $\dot{x} = f(x)$ with flow Φ_t , where x takes values on a finite dimensional smooth manifold M . Let $x^*(t)$ be a hyperbolic periodic solution with minimal period $T > 0$ and define the orbit $\gamma := \{x^*(t) \mid t \in [0, T]\}$. Prove or disprove, for our definition of hyperbolic diffeomorphisms, that γ is a hyperbolic set under the time- t map Φ_t :

(i) For $t = T$.

(ii) For $t > 0$ not rationally related to T .

Problem 32: [Smale Solenoid]

Realize the following sketch by a continuously differentiable map Φ of the (solid) 2-Torus $\mathbb{T}^2 := \text{disk} \times S^1$ into itself.



- (i) Give an example of Φ in suitable coordinates.
- (ii) Prove that the attractor $\mathcal{A} := \bigcap_{n=0}^{\infty} \Phi^n(\mathbb{T}^2)$ of Φ is hyperbolic.

Hint: Consider the coordinates (z, φ) with $|z| \leq 1$, $z \in \mathbb{C}$ and $\varphi \in \mathbb{R}/2\pi\mathbb{Z}$.