Homework Assignments
Dynamical Systems II
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http://dynamics.mi.fu-berlin.de/lectures/
due date: **Thursday, February 11, 2020, 16:00**

Problem 33: Consider a smooth diffeomorphism $\Phi : \mathbb{T}^2 \to \mathbb{T}^2$, C^1-close to A a linear, hyperbolic diffeomorphism of $\mathbb{T}^2$. Prove that Φ exhibits sensitivity to initial conditions.

Reminder: A map Φ is sensitive to initial conditions if there exists a $\varepsilon > 0$ such that for all $x_0 \in M$ and $\delta > 0$, there exist y_0 such that $||x_0 - y_0|| < \delta$ and a $n \in \mathbb{N}$ such that $||\Phi^n(x_0) - \Phi^n(y_0)|| > \varepsilon$.

Problem 34: Prove or disprove: The interior of any nonstationary periodic orbit of a planar C^1 vector field contains an equilibrium.

Problem 35: Discuss the structural stability of the following planar ODE phase diagrams in the regions shown. All the equilibria are assumed to be hyperbolic.
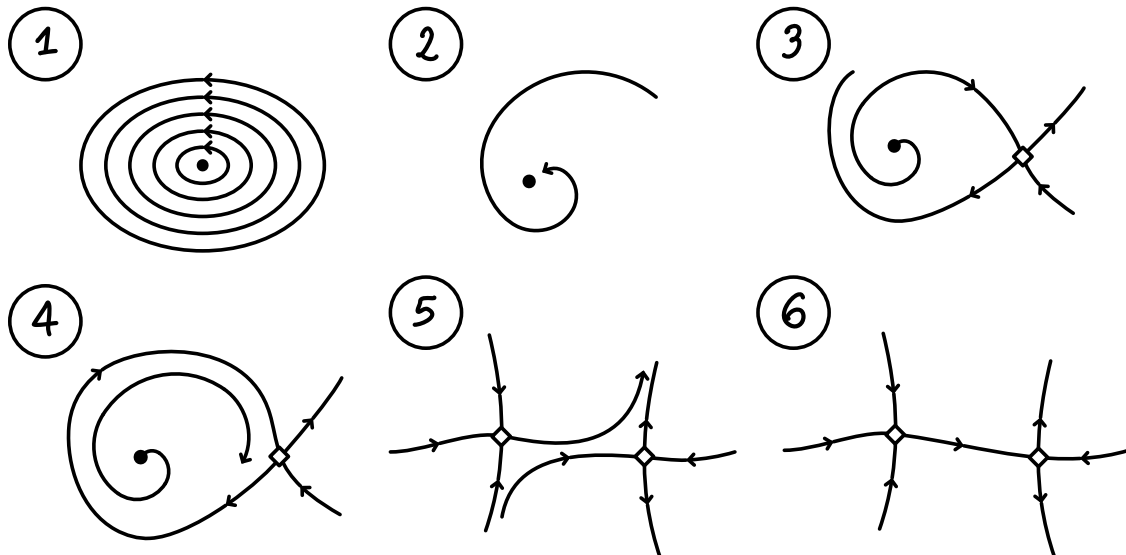
Problem 36: [Grobman & Hartman]

Consider the map $\Phi(x) = Ax + f(x) : \mathbb{R}^2 \to \mathbb{R}^2$ with A a hyperbolic matrix. Show that there exists $\varepsilon > 0$ such that for every $f \in C^0(\mathbb{R}^2, \mathbb{R}^2)$ such that f is bounded and

$$\sup_{(x,y)\in\mathbb{R}^2} \frac{|f(x) - f(y)|}{|x - y|} < \varepsilon$$

there is a homeomorphism H of $\mathbb{R}^2$ conjugating A to Φ.