Basic Questions of Dynamical Systems II

42. For ODEs, formulate the theorem on the existence of local/global center manifolds.

43. For diffeomorphisms, formulate the theorem on the existence of local/global center manifolds.

44. Discuss how cut-off functions help derive local from global center manifolds.

45. Is the global center manifold to a non-hyperbolic equilibrium of an almost linear C^1 vector field unique? How about the local center manifold?

46. Give an example of a planar vector field with a non-hyperbolic equilibrium for which the local center manifold is nonunique.

47. Give an example of a planar vector field with a non-hyperbolic equilibrium for which the 1-dimensional local center manifold is unique.

48. How are local center-stable and center-unstable manifolds for non-hyperbolic equilibria of autonomous vector fields defined? Give sufficient conditions for their existence.

49. What is the reduced ODE on a Lipschitz continuous local center manifold?

50. Let the global Lipschitz center manifold W^{c} of a bounded, almost linear C^1 vector field be given. In what sense is W^{c} unique? Prove your claim.

51. How can center manifolds assist in the study of bifurcation problems?

52. What is the "slaving" principle?

53. Sketch an example for the center manifold reduction of a saddle-node bifurcation. Comment on the unstable dimensions of the equilibrium branches.

54. Sketch an idea for a proof of the global center manifold theorem.

55. Give sufficient conditions for the center manifold to be attracting. Sketch a proof of your claim.

56. How can center manifolds assist in the study of singular perturbation problems?