Homework Assignments

Dynamical Systems III

Bernold Fiedler, Alejandro López Nieto

http://dynamics.mi.fu-berlin.de/lectures/

due date: Thursday, April 29, 2021, 18:00

Problem 1: Let Ψ_1 and Ψ_2 be smooth diffeomorphisms of $\mathbb{R}^N$. Show that the pullback $\Psi^* f := (D\Psi)^{-1} \circ f \circ \Psi$ satisfies

$$(\Psi_2 \circ \Psi_1)^* = \Psi_1^* \Psi_2^*.$$

Problem 2: Let $f : \mathbb{R}^N \to \mathbb{R}^N$ be a smooth map and $A, B \in \mathbb{R}^{N \times N}$. Show the following claims:

(i) For the adjoint $((\mathrm{ad}A)f)(x) := Af(x) - f'(x)Ax$, the matrix commutator $[A, B] := AB - BA$ satisfies

$$[\mathrm{ad}A, \mathrm{ad}B] = \mathrm{ad}[A, B].$$

(ii) For $A[f](x) := Af(x) - f(Ax)$, the matrix commutator satisfies

$$[A[\cdot], B[\cdot]] = AB[\cdot] - BA[\cdot].$$

Remark: Notice that both $\mathrm{ad}A$ and $A[\cdot]$ are linear maps in $C^\infty(\mathbb{R}^N, \mathbb{R}^N)$, therefore the commutator is well-defined.

Problem 3: Derive a formula for the dimension of $H_m(\mathbb{R}^N)$, the set of real homogeneous vector polynomials of degree m on N variables.

Problem 4:

Consider the following ODE on $\mathbb{R}^2$

$$\begin{aligned} \dot{x_1} &= x_2, \\ \dot{x_2} &= ax_1^2 + bx_1x_2, \end{aligned}$$

where $a, b \neq 0$. Show that one can assume without loss of generality $a = b = 1$, by choosing a suitable rescaling of the space and time variables x_1, x_2 and t.