Homework Assignments

Dynamical Systems III

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http://dynamics.mi.fu-berlin.de/lectures/

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Problem 37: Consider the periodically forced smooth ODE on $x \in \mathbb{R}^N$

$$\dot{x} = f(\lambda, t, x),$$

with parameters $\lambda \in \mathbb{R}^2$, 1-periodic forcing $f(\lambda, t+1, x) = f(\lambda, t, x)$, and a trivial solution$f(\lambda, t, 0) \equiv 0$, for all λ, t.
Give sufficient conditions on f under which subharmonic solutions with integer minimal period $q \geq 3$ bifurcate from the trivial equilibrium branch $x = 0$. Prove your claims.

Problem 38: Let ρ be a representation of a group H on $\mathbb{R}^N$ and consider an H-equivariant ODE

$$\dot{x} = f(x), \; x \in \mathbb{R}^N.$$

Assume there exists a homoclinic solution $x(t)$ to an equilibrium x^* such that the orbit of $x(t)$ is H-invariant, as a set. Prove or disprove:

(i) $\{x^*\}$ is H-invariant.

(ii) Each homoclinic point $x(t)$ is H-invariant.

(iii) If H is compact, H fixes every homoclinic point $x(t)$.

Problem 39: Consider the representation of $D_3 \times S^1$ on $\mathbb{C}^2$ given by

$$\begin{array}{rcl} \varphi(z_1, z_2) & = & (\mathrm{e}^{2\pi \mathrm{i}/3} z_1, \mathrm{e}^{-2\pi \mathrm{i}/3} z_2), \\ \sigma(z_1, z_2) & = & (z_2, z_1), \\ \vartheta(z_1, z_2) & = & (\mathrm{e}^{\mathrm{i}\vartheta} z_1, \mathrm{e}^{\mathrm{i}\vartheta} z_2). \end{array}$$

Here D_3, generated by the rotation φ, and the reflection σ is the symmetry group of the equilateral triangle, as in elementary geometry. Determine the isotropy lattice and the fixed-point subspaces.

Problem 40: Consider three oscillators $z_k \in \mathbb{C}$ in Hopf normal form, coupled to their neighbors "diffusively" via

$$\dot{z}_k = (\lambda + i + \gamma|z|_k^2)z_k + D(z_{k-1} - 2z_k + z_{k+1}), \qquad k \bmod 3, \quad \gamma, z_k \in \mathbb{C}, \quad \lambda, D \in \mathbb{R}.$$

Find the parameter values λ for which the linearizarion at $(z_1, z_2, z_3) = 0$ possesses a pair of complex conjugate eigenvalues. Discuss the occurrence of Hopf bifurcation and the spatio-temporal symmetry of at least one bifurcating solution.