Homework Assignments

Dynamical Systems III

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http://dynamics.mi.fu-berlin.de/lectures/

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Problem 21: Let ρ be a representation of the group Γ on a Banach space X, and consider a normal subgroup K of Γ. Show that ρ induces a representation of the factor group Γ/K on X_K, the subspace of K-fixed vectors.

Problem 22: Let $\gamma \in SO(3)$ act on the 6-dimensional space of real symmetric 3×3 matrices A via $\rho(\gamma)A := \gamma A \gamma^T$. Show that:

(i) ρ is a representation.

(ii) ρ is not irreducible.

(iii) ρ is irreducible on the subspace of matrices A with zero trace $\mathrm{tr}(A) = 0$.

Problem 23: Consider the space $L^2(S^1)$, $S^1 = \mathbb{R}/2\pi\mathbb{Z}$, of 2π-periodic, square-integrable, complex-valued funtions. The canonical representation of the group $SO(2) = S^1$ on $L^2(S^1)$ is given by

$$(\gamma f)(x) = f(\gamma + x).$$

Determine all irreducible complex subspaces of $L^2(S^1)$.

Hint: Consider the unitary Fourier transform $T : L^2(S^1) \to \ell_2(\mathbb{C})$ given by

$$a_k := \frac{1}{2\pi} \int_0^{2\pi} e^{-ikt} f(t) \mathrm{d}t$$

and the Fourier series

$$f(t) = \sum_{k \in \mathbb{Z}} a_k e^{ikt}.$$

Problem 24: Consider the splitting of a Banach space $X = U \oplus V$ into closed subspaces, with associated continuous projection $P : X \to U$ onto U. Let $\rho(\gamma)x = \gamma x$ be a representation of a finite group Γ on X, $|\Gamma| = N$ and assume $U = \gamma U$ for all $\gamma \in \Gamma$.

Show that:

(i) $\tilde{P}x := \frac{1}{N}\sum_{\gamma\in\Gamma} \gamma^{-1}P\gamma x$ defines a Γ-equivariant map.

(ii) $\tilde{P}$ is a projection into U, i.e. $\tilde{P}^2 = \tilde{P}$.

(iii) $\ker\tilde{P} = V$, implies that V is Γ-invariant.