

Übungen zur Vorlesung

Analysis II

Sommersemester 2022

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<http://dynamics.mi.fu-berlin.de/lectures/>

Due date: Wednesday, 11.05.2022, 17:00.

Solutions in German or English, please.

Problem 5: Compute

$$\int_1^2 \frac{1}{t^2} dt$$

by approximating the integrand $f(t) = 1/t^2$ through step functions (Treppenfunktionen).

Hint: Consider the partition given by $1 = a_0 < a_1 < \dots < a_n = 2$, where $a_k = 2^{k/n}$.

Problem 6: Show that the map

$$\|f\|_1 := \int_a^b |f(t)| dt,$$

defines a norm on the space $\mathcal{R}([a, b], \mathbb{R})$ of regulated functions (Regelfunktionen). Is $\mathcal{R}([a, b], \mathbb{R})$ with $\|\cdot\|_1$ a Banach space?

Problem 7: Consider a regulated function $f : [a, b] \rightarrow \mathbb{R}$. Is the primitive function (Stammfunktion) $F(x) := \int_a^x f(t) dt$ a regulated function?

Problem 8: Let $f, g : [0, 1] \rightarrow [0, 1]$ be regulated functions. Prove or disprove

- (i) $g \circ f : [0, 1] \rightarrow [0, 1]$ is a regulated function.
- (ii) If g is continuous, then $g \circ f : [0, 1] \rightarrow [0, 1]$ is a regulated function.
- (iii) If f is continuous, then $g \circ f : [0, 1] \rightarrow [0, 1]$ is a regulated function.

Here \circ denotes the composition $(g \circ f)(x) = g(f(x))$.