Übungen zur Vorlesung

Analysis II

Sommersemester 2022

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http://dynamics.mi.fu-berlin.de/lectures/

Due date: Wednesday, 18.05.2022, 17:00.

Solutions in German or English, please.

Problem 9: Show that the formula for the area of the rectangle $A(a,b) = ab$, where a and b denote the length of the sides, is the only continuous function $A : \mathbb{R}_+^2 \to \mathbb{R}_+$ such that for all $a, a_1, a_2, b > 0$

(i) $A(a,b) = A(b,a)$.

(ii) $A(a_1 + a_2, b) = A(a_1, b) + A(a_2, b)$.

(iii) $A(1,1) = 1$.

Interpret the assumptions (i)–(iii).

Problem 10: Show that:

(i) Given an *odd* function $f : [-a,a] \to \mathbb{R}$, i.e., such that $f(-x) = -f(x)$ for all $x \in [-a,a]$. Show

$$\int_{-a}^{a} f(x)\,\mathrm{d}x \;=\; 0.$$

(ii) Given a *periodic* function $g : \mathbb{R} \to \mathbb{R}$ with period $p > 0$, i.e., such that for all $x \in \mathbb{R}$ we have that $g(x+p) = g(x)$. Then for all $a \in \mathbb{R}$

$$\int_{a}^{a+p} g(x)\,\mathrm{d}x \;=\; \int_0^p g(x)\,\mathrm{d}x.$$

Problem 11: For all integers k, l, determine the so-called *orthogonality relations*

$$\frac{1}{\pi}\int_0^{2\pi} \sin(kx)\sin(lx)\,\mathrm{d}x, \qquad \frac{1}{\pi}\int_0^{2\pi} \sin(kx)\cos(lx)\,\mathrm{d}x, \qquad \frac{1}{\pi}\int_0^{2\pi} \cos(kx)\cos(lx)\,\mathrm{d}x.$$

Problem 12: Determine four of the following integrals

(i) $\displaystyle\int \sqrt{t}\,\ln t\,\mathrm{d}t$;

(ii) $\displaystyle\int \frac{\mathrm{d}t}{\cos t+\sin t}$;

(iii) $\displaystyle\int \sqrt{\tan\frac{t}{2}}\,\mathrm{d}t$;

(iv) $\displaystyle\int t\sqrt{1+t}\,\mathrm{d}t$;

(v) $\displaystyle\int \mathrm{e}^{\sqrt{t}}\,\mathrm{d}t$;

(vi) $\displaystyle\int \frac{1}{t^4+1}\,\mathrm{d}t$;

(vii) $\displaystyle\int t^2(\ln t)^2\,\mathrm{d}t$;

(viii) $\displaystyle\int t^2\mathrm{e}^{-t}\,\mathrm{d}t$.