

Übungen zur Vorlesung

Analysis II

Sommersemester 2022

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<http://dynamics.mi.fu-berlin.de/lectures/>

Due date: Wednesday, 01.06.2022, 17:00.

Solutions in German or English, please.

Problem 17: Let $f : [0, 1] \rightarrow [0, 1]$ be Riemann integrable function and let $g : [0, 1] \rightarrow [0, 1]$ be continuous, prove or disprove

(i) $g \circ f$ is Riemann integrable.

(ii) $f \circ g$ is Riemann integrable.

Problem 18:

(i) Consider a continuous function $f : [a, b] \rightarrow \mathbb{R}$ such that $f \geq 0$ and

$$\int_a^b f(t) dt = 0.$$

Show that $f(t) = 0$ for all $t \in [a, b]$.

(ii) Consider a continuous function $f : [a, b] \rightarrow \mathbb{R}$ such that for all $g : [a, b] \rightarrow \mathbb{R}$ continuous

$$\int_a^b f(t)g(t) dt = 0.$$

Show that $f(t) = 0$ for all $t \in [a, b]$.

Problem 19: Use Taylor series to derive an expansion for the quantity

$$\int_0^1 t^t dt.$$

Problem 20: Discuss the convergence of the following improper integrals and, if possible, determine their value

(i) $\int_e^\infty \frac{dt}{t \log t}$;

(ii) $\int_0^\infty \frac{dt}{t(t^2 + 1)}$;

(iii) $\int_{-\infty}^0 e^t \cos t dt$;

(iv) $\int_0^1 \frac{dt}{\sqrt{t(1-t)}}$.

Hint: $t = \sin^2 \varphi$.