Homework

V19028: Dynamical Systems II

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Problem 1: Let $0 < \beta < 1$ be irrational and

$$s_n := \text{sign}(\sin(n\pi\beta)), \qquad n = 1, 2, 3, ...$$

The sequence

$$w_n := |s_n - s_{n+1}|/2$$

detects the sign changes of the sequence s_n. Prove:

$$\lim_{N\to\infty} \frac{1}{N} \sum_{n=1}^{N} w_n = \beta.$$

Free extra: Is it possible to recover a rational number β from the sequence s_n?

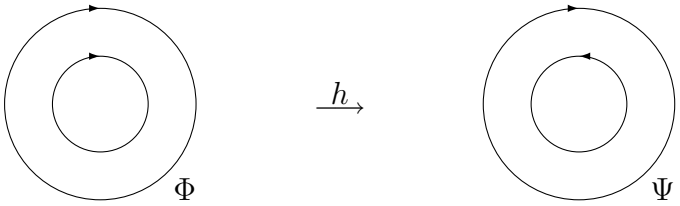
Problem 2: Let $f : S^1 \to S^1$ be a homeomorphism of the circle that reverses orientation, i.e. the induced map $F : \mathbb{R} \to \mathbb{R}$ on the covering space $\mathbb{R}$ satisfies $F(x+2\pi) = F(x) - 2\pi$ for all $x \in \mathbb{R}$.

Prove or disprove: f has a fixed point.

Problem 3: As stated in class, each flow φ_t on the 2-torus without equilibria and periodic orbits possesses a global transverse section S, i.e. a closed curve that intersects every trajectory and is everywhere transverse to the vector field.

Find an example of a flow φ_t on the 2-torus with (two or more) periodic orbits — but without equilibria — such that no global transverse section S exists.

Problem 4: Consider dynamical systems on a ring R, bounded by two periodic orbits.



Assume that there are no periodic orbits or equilibria in the interior of R.

Prove or disprove: there exists flows Φ_t and Ψ_t, as sketched above, which are

(i) $\mathcal{C}^0$ orbit equivalent, i.e. there exists a homeomorphism $h : R \to R$ mapping orbits of Φ onto orbits of Ψ;

(ii) $\mathcal{C}^0$ flow equivalent, i.e. there exists a homeomorphism $h : R \to R$ such that $h \circ \Phi_t = \Psi_t \circ h$.