Homework

V19028: Dynamical Systems II

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Problem 29: Consider the Hénon map

$$\begin{aligned} x_{j+1} &= 1 - \alpha x_j^2 + \beta y_j, \\ y_{j+1} &= x_j. \end{aligned}$$

Find a horseshoe for $1 \ll \alpha$ and $0 < \beta \ll 1$.

Hint: $Q = [-0.1, 0.1] \times [-1, 1]$.

Problem 30: A measure of complexity of a map Φ is the *topological entropy* h: Let $N(n)$ be the number of periodic points of Φ with (not necessarily minimal) period n. Then the entropy is defined as

$$h := \limsup_{n\to\infty} \frac{\log N(n)}{n}.$$

Calculate the entropy h of the shift on m symbols. Prove that every iteration Φ containing a shift (i.e. with an invariant set I such that $\Phi|_I$ is conjugate to a shift on m symbols) has positive topological entropy.

Problem 31: Consider the bouncing-ball map f,

$$\begin{aligned} \Phi_{j+1} &= \Phi_j + v_j, \\ v_{j+1} &= \alpha\, v_j - \gamma \cos(\Phi_j + v_j), \end{aligned}$$

discussed in class, with $\alpha = 1$ and $\gamma > 0$. Choose γ large enough and find a "horseshoe" for $(\Phi, v) \in (0, 4\pi) \times \mathbb{R}$ giving rise to an invariant set I such that $f|I$ in conjugate to the shift on 4 symbols.

Free extra: Find a shift on m symbols for every $m \geq 2$.

Problem 32: Consider again the bouncing-ball map f:

$$\begin{aligned} \Phi_{j+1} &= \Phi_j + v_j, \\ v_{j+1} &= \alpha\, v_j - \gamma \cos(\Phi_j + v_j) \end{aligned}$$

with $(\Phi, v) \in S^1 \times \mathbb{R} = (\mathbb{R}/(2\pi\mathbb{Z})) \times \mathbb{R}$.

What is the (non-unique!) lift $F : \mathbb{R}^2 \to \mathbb{R}^2$ of f? Choose a suitable lift and modify the horseshoe construction done in class (or in the solution to the previous problem) such that $v > 0$ on the induced invariant set.

Free extra: Give a physical interpretation.