Homework

# V19028: Dynamical Systems II

Bernold Fiedler, Stefan Liebscher

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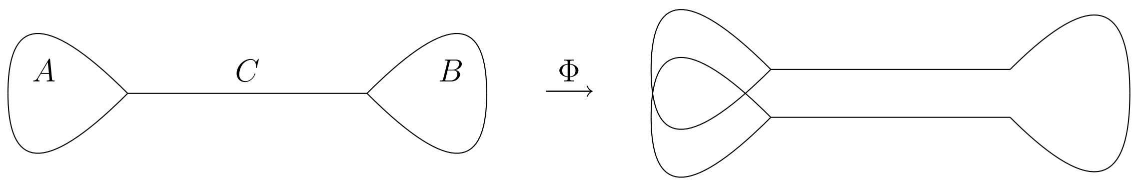
**Problem 37:** Try to realize the following graph by a continuous map $\Phi$ (similar to the case of the Plykin attractor)

$$\begin{array}{rcl} A & \to & A \\ B & \to & A \\ C & \to & C + B - C \end{array}$$




Why does this construction not yield a hyperbolic attractor?

**Problem 38:** Consider the Plykin attractor defined in class as the $\omega$-limit set of an initial domain $M = A \cup B \cup C \cup D$ under a diffeomorphism $\Phi$,

$$P := \bigcap_{n=0}^{\infty} \Phi^n M,$$

The domain $M$ is compact, connected, and path-connected. The same holds true for all iterates $\Phi^n M$. Prove or disprove:

(i) The Plykin attractor is connected.

(ii) The Plykin attractor is path-connected.

*Reminder:* A set $X$ is called connected if it cannot be split into two nontrivial open sets, i.e. $U \subset \mathbb{R}^2$ is connected if for all open sets $U, V \subset \mathbb{R}^2$

$$\Big( X \subset U \cup V \text{ and } U \cap V = \emptyset \implies X \subset U \text{ or } X \subset V. \Big)$$

A set $X$ is called path-connected if every two points in X can be connected by a continuous path, i.e.

$$\forall x, y \in X \; \exists f : [0,1] \to X \text{ continuous } \; : \; f(0) = x \text{ and } f(1) = y.$$

**Problem 39:** Consider matrices $A \in SL(2, \mathbb{Z})$ with integer coefficients and determinant 1. These define linear orientation- and area-preserving maps on the plane.

(i) Prove that every $A \in SL(2, \mathbb{Z})$ defines a diffeomorphism of the torus $T^2 = \mathbb{R}^2/\mathbb{Z}^2$.

(ii) Which $A \in SL(2, \mathbb{Z})$ are structurally stable?

*Free extra:* What happens for matrices with determinant $-1$?

**Problem 40:** Consider the map

$$\Phi \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 & 2 \\ 4 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \frac{1}{1000} \begin{pmatrix} \sin(2\pi x)\cos(2\pi y) \\ \cos(2\pi x)\sin(2\pi y) \end{pmatrix}$$

on the torus $T^2 = \mathbb{R}^2/\mathbb{Z}^2$. How many homoclinic orbits does the point $(0, 0) \in T^2$ have under iterations of $\Phi$ (zero, finitely many, countably many, more than countably many)?