Homework

V19263: Basic Course Dynamical Systems

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Problem 21: Consider a vector field on $\mathbb{R}^n$ with equilibrium at the origin,

$$\dot{x} = f(x), \qquad f(0) = 0.$$

Let $A = Df(0)$ denote the linearization of the vector field at the origin.

Determine all Wronski-Matrices $W(t, t_0)$ to the initial value $x_0 = 0$.

Problem 22: Let $A = (a_{ij})_{1 \le i,j \le n}$ be a real $(n \times n)$-matrix.

Prove: If the coefficients of the matrix e^{At} are non-negative for all $t \ge 0$ then $a_{ij} \ge 0$ for all $i \ne j$.

Problem 23: Calculate the solutions of the following linear differential equations

$$\text{(i)} \qquad \dot{x} = \begin{pmatrix} -3 & 0 & 2 \\ -1 & -3 & 5 \\ -1 & 0 & 0 \end{pmatrix} x, \qquad x(0) = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix}$$

$$\text{(ii)} \qquad \dot{x} = \begin{pmatrix} 3 & 1 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & -2 \end{pmatrix} x, \qquad x(0) = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}.$$

Problem 24: We want to understand the damped linear pendulum

$$\ddot{x} + \nu\dot{x} + \omega^2 x = 0$$

with parameters $\boldsymbol{\nu, \omega > 0}$ and initial conditions $x(0) = 0$, $\dot{x}(0) = 1$.

(i) Determine the explicit solution of the given initial-value problem for all ν, ω. Sketch phase portraits and a diagram of the (ν, ω)-plane of different qualitative behavior.

(ii) How does the phase portrait change at the boundary between different zones in the (ν, ω)-plane, for instance due to a change of the damping? How do you reconcile the discontinuities in the phase portraits of the Jordan normal form with differentiable dependence of the flow on the parameters ν, ω?