Homework

V19263: Basic Course Dynamical Systems

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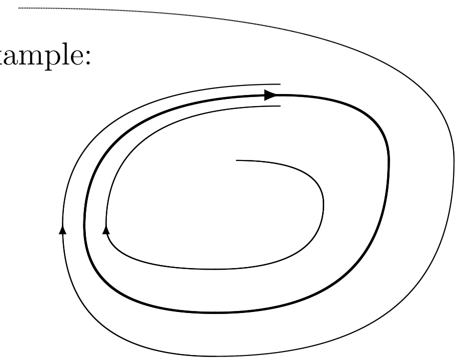
Problem 29: Consider a periodic orbit Γ of a flow φ in $X = \mathbb{R}^n$ and a neighborhood U of Γ in X such that each trajectory $\gamma(x_0)$, $x_0 \in U$, converges to Γ as $t \to \infty$.

Prove or disprove: Every first integral of φ is constant in U.

Example:



Problem 30: [see Arnol'd, 2.4.5] Determine all $k \in \mathbb{R}$ such that the system

$$\begin{aligned} \dot{x_1} &= x_1 \\ \dot{x_2} &= kx_2 \end{aligned}$$

possesses a non-constant first integral for $(x_1, x_2) \in \mathbb{R}^2$.

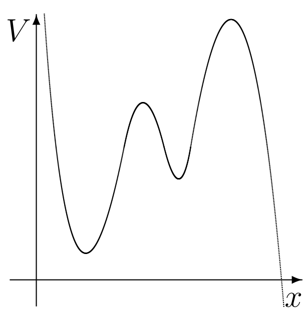
Problem 31: Sketch the phase portraits of $\ddot{x} + V'(x) = 0$,

(i) for the Kepler problem,

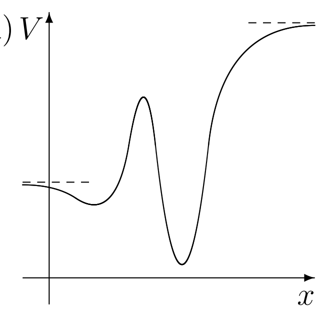
$$V(x) = -\frac{1}{x} + C\frac{1}{x^2},$$

$C > 0$, $x > 0$

(ii)



(iii)



Pay attention to saddle equilibria, homoclinic orbits, and asymptotic behavior at infinity.

Problem 32: Consider a radially symmetric vector field in the plane,

$$\begin{aligned} \dot{x} &= f(x^2+y^2)x - g(x^2+y^2)y, \\ \dot{y} &= g(x^2+y^2)x + f(x^2+y^2)y. \end{aligned}$$

(i) Find an Euler multiplier that turns it into a divergence-free vector field.

(ii) Find an example of the above form that does *not* possess a nontrivial First Integral.

(iii) What is wrong?