Homework

V19263: Basic Course Dynamical Systems

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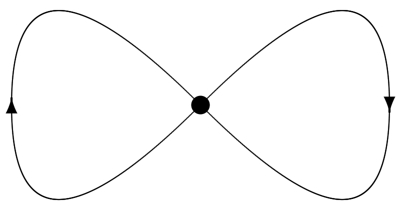
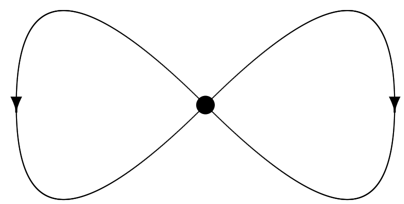
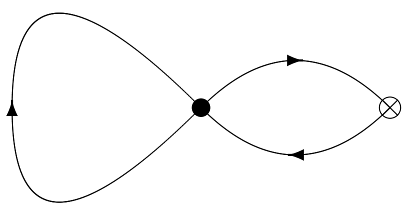
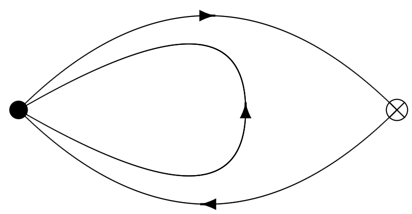
due date: Tuesday, June 26, 2007

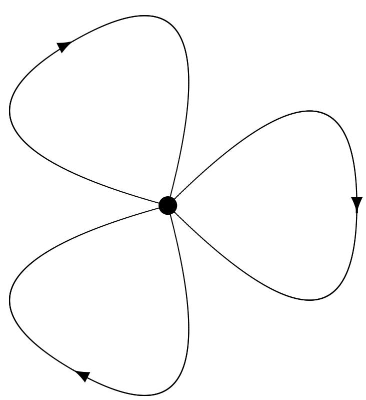
Problem 33: Which of the following sets are possible ω-limits of a (single) trajectory of some planar flow? Which of the sets cannot occur as ω-limits (of a single trajectory)? Justify your claims, without providing explicit vector fields.

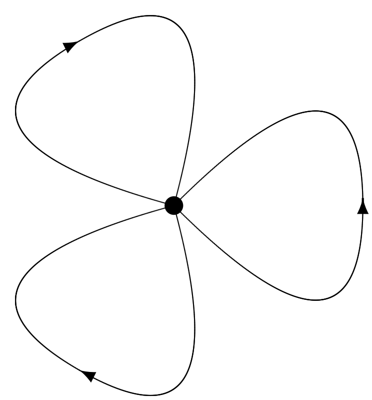
(i)


(ii)


(iii)


(iv)


(v)


(vi)


Discs ● denote equilibria (of any type) and crossed out circles $\otimes$ denote hyperbolic saddles.

Problem 34: A (point-sized) professor — trying to escape from his office hour — starts from his office at the origin $(x, y) = (0, 0)$ of the plane $\mathbb{R}^2$ and runs along the positive x-axis (i.e. Arnimallee) with speed 1. At the same moment a (point-sized) student — with loads of innocent questions concerning the homework assignment — starts at the point $(x, y) = (0, 1)$ and chases the professor. The student has the same speed 1 and always runs directly towards the professor.

How close does the student get? Will her/his questions ever be answered?

Hint: Use appropriate coordinates (e.g. r = distance of student and professor, φ = angle of the x-axis with the connecting line of both persons) and solve the resulting system by separation of variables.

Problem 35: Consider the pendulum equation

$$\ddot{x} + g(x) = 0$$

for a continuous <u>odd</u> function g with $g(x) \cdot x > 0$ for all $x \neq 0$. Let $p(g, a) > 0$ be the minimal period of the solution to the initial value $x(0) = a > 0$, $\dot{x}(0) = 0$.

Prove:

(i) If $g_1(x) < g_2(x)$ for all $x > 0$ then $p(g_1, a) > p(g_2, a)$ for all $a > 0$.

(ii) If $x \mapsto g(x)/x$ is strictly monotonically decreasing for $x > 0$, then $a \mapsto p(g, a)$ is strictly monotonically increasing for $a > 0$.

Hint: $y(t) := \frac{a_1}{a_2} x(t)$ solves the equation $\ddot{y} + \tilde{g}(y) = 0$ with $\tilde{g}(y) := \frac{a_1}{a_2} g(\frac{a_2}{a_1} y)$.

Problem 36: In class we proved that for arbitrary flows on $\mathbb{R}^n$ and every <u>bounded</u> (forward) trajectory $\gamma^+(x_0)$ the ω-limit $\omega(x_0)$ is non-empty, invariant, compact, and connected.

Drop the requirement of boundedness. For each row of the table

non-empty	invariant	compact	connected
no	no	no	no
no	no	no	yes
no	no	yes	no
no	no	yes	yes
no	yes	no	no
no	yes	no	yes
no	yes	yes	no
no	yes	yes	yes
yes	no	no	no
yes	no	no	yes
yes	no	yes	no
yes	no	yes	yes
yes	yes	no	no
yes	yes	no	yes
yes	yes	yes	no
yes	yes	yes	yes

either find an example of an <u>unbounded</u> trajectory with ω-limit set with the prescribed properties or prove that no such example exists.