Homework

V19263: Basic Course Dynamical Systems

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Problem 41: Let $X \subset \mathbb{R}^2$ be a circular disc with ℓ disjoint circular holes and f a continuously differentiable vector field on X with $\operatorname{div} f > 0$.

Prove: the corresponding flow contains at most ℓ periodic orbits in X.

Problem 42: Prove or disprove the theorem of POINCARÉ & BENDIXSON for flows

(i) on the sphere S^2,

(ii) on the torus T^2.

Problem 43: Choose at least one variant:

(A) In a bounded planar classroom, a professor chases a smart student. Both run at constant velocity 1 in arbitrary (and variable) direction. Can the professor reach the student?

(B) The student runs along a circle, with constant velocity 1. The professor starts at the center and keeps running towards the momentary position of the student, at constant velocity $p < 1$. What is the professor's ω-limit set? Is the ω-limit set stable? What happens for $p = 1$, $p > 1$?

Hint (B): Useful coordinates are the angle between origin and professor, as seen from the student, and the distance between student and professor.

Free extra: Chase the professor.

Problem 44: Let f be a planar C^1 vector field and let the forward orbit $\gamma_+(x_0)$ of some initial condition $x_0 \in \mathbb{R}^2$ be bounded. Assume that $\omega(x_0)$ is neither an equilibrium nor a periodic orbit.

Prove: $\omega(x_0) = E \cup H$ is the union of

(i) a set E containing only equilibria and

(ii) a set H containing only homoclinic or heteroclinic orbits.

Can H contain countably/uncountably many orbits?