The multiplicity of positive solutions
to a quasilinear Neumann problem with critical exponent

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We consider the following boundary value problem:

\[
\begin{aligned}
\Delta_p u & := \text{div}(|\nabla u|^{p-2} \nabla u) = |u|^{p-2} u \quad \text{in } B_R \\
|\nabla u|^{p-2}\langle \nabla u; n \rangle &= |u|^{q-2} u \quad \text{on } \partial B_R,
\end{aligned}
\]  

(1)

where $B_R$ is the ball of radius $R$ in $\mathbb{R}^n$, $n$ is a unit exterior normal vector, $1 < p \leq \frac{n+1}{2}$, $q = \frac{(n-1)p}{(n-p)}$ is the critical Sobolev exponent for the trace embedding.

We establish the multiplicity of positive solutions for the problem (1). Namely, we prove that for every $m > 0$ there is $R_0 = R_0(m, p)$, such that for all $R > R_0$ the problem (1) has at least $m$ non-equivalent positive solutions.

For subcritical exponents effect of multiplicity of solutions was studied in [1] and [2]. The similar multiplicity effect for the Dirichlet boundary value problem was established in a number of papers beginning with the pioneering work [3].

References

