The multiplicity of positive solutions to a quasilinear Neumann problem with critical exponent

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We consider the following boundary value problem:

$$\begin{cases} \Delta_p u := \operatorname{div}(|\nabla u|^{p-2}\nabla u) = |u|^{p-2}u \quad \text{in } B_R \\ \\ |\nabla u|^{p-2}\langle \nabla u; \mathbf{n}\rangle = |u|^{q-2}u \quad \text{on } \partial B_R, \end{cases} \tag{1}$$

where B_R is the ball of radius R in $\mathbb{R}^n$, $\mathbf{n}$ is a unit exterior normal vector, $1 < p \leq \frac{n+1}{2}$, $q = \frac{(n-1)p}{(n-p)}$ is the critical Sobolev exponent for the trace embedding.

We establish the multiplicity of positive solutions for the problem (1). Namely, we prove that for every $m > 0$ there is $R_0 = R_0(m, p)$, such that for all $R > R_0$ the problem (1) has at least m non-equivalent positive solutions.

For subcrtitical exponents effect of multiplicity of solutions was studied in [1] and [2]. The similar multiplicity effect for the Dirichlet boundary value problem was established in a number of papers beginning with the pioneering work [3].

References

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