Multiplicity of positive solutions to the boundary value problems with fractional Laplacians

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In 1984 Coffman in the paper [1] showed that for $n = 2$ boundary value problem with Laplacian

$$-\Delta u = u^{q-1} \quad \text{in} \quad \Omega = B_{R+1} \setminus B_R, \quad u|_{\partial\Omega} = 0 \quad u \geq 0 \tag{1}$$

has the so-called "multiplicity effect": for each $N \in \mathbb{N}$ there exists R_0 such that for all $R \geq R_0$ problem (1) has N different positive solutions. The same effect also was achieved in [2], [3] for the Laplacian in case of $n \geq 3$ and p-Laplacian [4], [5].

We establish the "multiplicity effect" for the problem

$$(-\Delta)^s u = u^{q-1} \quad \text{in} \quad \Omega_R = B_{R+1} \setminus B_R \in \mathbb{R}^n, \quad u \in \widetilde{H}^s(\Omega_R) \quad u \geq 0 \tag{2}$$

where $0 < s \leq 1$, $2 < q < \frac{2n}{(n-2s)_+}$, and the space $\widetilde{H}^s(\Omega_R)$ is defined by

$$\widetilde{H}^s(\Omega_R) = \left\{ u \in H^s(\mathbb{R}^n) \mid \operatorname{supp}(u) \subset \overline{\Omega}_R \right\}.$$

$(-\Delta)^s$ in (1) stands either for Navier-type or for Dirichlet-type fractional Laplacian.

References

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