DYNAMICS PROPERTIES OF SECOND-ORDER EQUATIONS WITH LARGE DELAY

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We'll study equation

$$\frac{d^2x}{dt^2} + \sigma\frac{dx}{dt} + x = ax(t-T) + F(x(t-T))$$

in conditions $T \gg 1$. Let's consider $\sigma > 0$ (this guarantee decay of all solutions when right side of equation is equal to zero).

Critical cases of equilibrium state stability problem (and bifurcations) take place when a is near to $\pm a(\sigma)$. $a(\sigma)$ defined as

$$a(\sigma) = \begin{cases} 1, & \sigma > \sqrt{2}, \\ \sigma\sqrt{1-\sigma^2/4}, & \sigma < \sqrt{2}. \end{cases}$$

First, let $\sigma > \sqrt{2}$. So critical value for a is $a = \pm(1+\mu a_1)$ $(0 < \mu \ll 1)$. This situation is like the same situation for the first-order equations [1]. For example, if $a = -1 - \mu a_1$ then normal form is (see [2])

$$\frac{\partial u}{\partial \tau} = \omega^2\frac{\sigma^2-2}{2}\frac{\partial^2 u}{\partial r^2} + a_1u + fu^3$$

with antiperiodic boundary conditions

$$u(\tau, r+1) = -u(\tau, r).$$

Here parameter ω is arbitrary, f is some constant.

Let now $0 < \sigma < \sqrt{2}$ and $a = \pm a(\sigma)(1+\mu a_1)$. This situation is more interesting because dynamics is more rich. Normal form is (see [2])

$$\frac{\partial u}{\partial \tau} = \omega^2\frac{2-\sigma^2}{a(\sigma)^2}\frac{\partial^2 u}{\partial r^2} + a_1u + d|u|^2u \tag{1}$$

with periodic boundary conditions

$$u(\tau, r) = u(\tau, r+1). \tag{2}$$

As in first case, ω is arbitrary parameter, d is some complex constant.

References

[1] Kashchenko I.S. Asymptotic analysis of the behavior of solutions to equations with large delay // Doklady Mathematics, 2008, Vol. 78, No. 1, pp. 570–573.

[2] Kashchenko I.S. Local dynamics of equations with large delay // Compute Math. and Math. phys., 2008, Vol 48, No 12, pp. 2141–2150.