

Compactness and Structural Stability of Nonlinear Flows

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After the seminal works [1],[2] of Brezis, Ekeland, Nayroles and Fitzpatrick, maximal monotone operators $\alpha : V \rightarrow \mathcal{P}(V')$ (V being a Banach space) and flows of the form

$$\frac{du}{dt} + \alpha(u) \ni h \quad \text{in } V', \text{ a.e. in }]0, T[, \quad u(0) = u^0 \quad (1)$$

can be formulated as a minimization principle, even if α is not a subdifferential.

On this basis, De Giorgi's notion of Γ -convergence may be applied to the analysis of monotone inclusions. Compactness and structural stability of the Cauchy problem are then studied, with respect to arbitrary variations not only of the datum $h \in L^2(0, T; V')$, but also of the operator α . This suggests the use of an exotic nonlinear topology of weak type, see [4]–[7].

The operator α may also be assumed to be a multivalued pseudo-monotone operator, e.g.:

$$\alpha(u) = -\nabla \cdot \vec{\gamma}(u, \nabla u) \quad \forall u \in W_0^{1,p}(\Omega),$$

with $\vec{\gamma}$ lower semicontinuous (as a multivalued operator) w.r.t. the first argument, and maximal monotone w.r.t. the second one.

These results can be extended in several directions, and can be applied to nonlinear either stationary or evolutionary PDEs. In particular, one can deal with doubly-nonlinear parabolic inclusions of the form

$$D_t \partial \varphi(u) + \alpha(u) \ni h \quad \text{or} \quad \alpha(D_t u) + \partial \varphi(u) \ni h, \quad (2)$$

with α as above and φ convex and lower semicontinuous.

These recent results rest upon a novel notion of *evolutionary Γ -convergence* of weak-type.

References

- [1] H. BREZIS, I. EKELAND: *Un principe variationnel associé à certaines équations paraboliques. I. Le cas indépendant du temps, II. Le cas dépendant du temps.* C. R. Acad. Sci. Paris Sér. A-B **282** (1976) 971–974, and *ibid.* 1197–1198
- [2] B. NAYROLES: *Deux théorèmes de minimum pour certains systèmes dissipatifs.* C. R. Acad. Sci. Paris Sér. A-B **282** (1976) A1035–A1038
- [3] S. FITZPATRICK: *Representing monotone operators by convex functions.* Workshop/Miniconference on Functional Analysis and Optimization (Canberra, 1988), 59–65, Proc. Centre Math. Anal. Austral. Nat. Univ., 20, Austral. Nat. Univ., Canberra, 1988
- [4] A. VISINTIN: *Extension of the Brezis-Ekeland-Nayroles principle to monotone operators.* Adv. Math. Sci. Appl. **18** (2008) 633–650
- [5] A. VISINTIN: *Variational formulation and structural stability of monotone equations.* Calc. Var. Partial Differential Equations **47** (2013), 273–317
- [6] A. VISINTIN: *On Fitzpatrick's theory and stability of flows.* Rend. Lincei Mat. Appl. **27** (2016) 1–30
- [7] A. VISINTIN: *Structural compactness and stability of pseudo-monotone flows.* (forthcoming).