Morse saddle. Obviously there is only one bounded face F: the interior of the n-gon. The boundary ∂F is the 1-skeleton G.
To design the associated n-gon Sturm attractor we follow the recipe at the end of section 2.1: from orientations via ZS-Hamiltonian pairs and Sturm permutations to Sturm attractors.
The loop-free orientations of G, without di-sources and di-sinks other than the Morse sinks $\underline{v}$ and $\bar{v}$, are characterized by the positions of $\underline{v}$ and $\bar{v}$ along the boundary n-gon ∂F: the two n-gon arcs between $\underline{v}$ and $\bar{v}$ are oriented from $\underline{v}$ to $\bar{v}$. Without loss of generality we label

$$\underline{v} = v_1, \qquad \bar{v} = v_{2m+1} \tag{2.23}$$

for some $1 \leq m < n$. See figure 2.5 for the unique ZS-Hamiltonian paths which result from this orientation.
According to the recipe of section 2.1, the Sturm permutation $\pi = \pi_{n,m}$ is then given by (2.3):

$$\pi = \begin{pmatrix} 1 & 2 & \ldots & 2(n-m) & 2(n-m)+1 & 2(n-m)+2 & \ldots & 2n & 2n+1 \\ 1 & 2m+2 & \ldots & 2n & 2m+1 & 2 & \ldots & 2m & 2n+1 \end{pmatrix}. \tag{2.24}$$

See figure 2.6 for the associated shooting curve and the Morse indices i_k of $\pi_{n,m}$. This completes the design of all n-gon Sturm attractors.

3 Specifics: 37 planar Sturm attractors with up to 11 equilibria

We apply theorems 1.1 and 1.2 to enumerate all planar Sturm attractors with up to 11 hyperbolic equilibria, in this section. We eliminate orientation isomorphic duplicates and the trivial substitution equivalences generated by $x \mapsto -x$, and by $v \mapsto -v$; see section 2.2 above. We use the duality pairings of section 2.4 to effectively cut the total number of cases in half.
Except for one indecomposable self-dual case, we do not follow the general design recipe detailed at the end of section 2.1. Instead, we proceed from the n-gons of section 2.6 by the attractor stacking and face gluing decompositions of sections 2.3 and 2.5, to inductively exhaust all possibilities.
For each nontrivial example we present the oriented 1-skeleton $G = \mathcal{C}_f^1$ of the connection graph, the oriented dual 1-skeleton $G^* = \mathcal{C}_{f^*}^1$ as introduced in section 2.4, the unique boundary ZS-Hamiltonian pair (h_0, h_1), and the associated Sturm permutation π.
Let μ_i count the number of equilibria with Morse index $i \in \{0, 1, 2\}$. We then enumerate the 37 connection graphs $\mathcal{C}$ with up to 11 equilibria, and up to trivial substitution equivalences,

(a) by increasing the odd number $N = 3, \dots, 11$ of equilibria, and

(b) by increasing the number μ_2 of faces.

Duality is utilized as follows. Obviously the total vertex count is

$$\mu_0 + \mu_1 + \mu_2 = N \tag{3.1}$$

for plane attractors. On the other hand, μ_0 counts vertices, μ_1 edges, and μ_2 faces of the plane 1-skeleton $G = \mathcal{C}_f^1$. By the Euler characteristic (1.10)

$$\mu_0 - \mu_1 + \mu_2 = 1, \quad \text{and hence} \tag{3.2}$$

$$\begin{aligned} \mu_0 &= \frac{1}{2}(N+1) - \mu_2 \\ \mu_1 &= \frac{1}{2}(N-1). \end{aligned} \tag{3.3}$$

Let μ_j^* denote the Morse counts for the dual graph G^*. Then

$$\begin{aligned} \mu_0^* &= \mu_2 + 2 \\ \mu_1^* &= \mu_1 \\ \mu_2^* &= \mu_0 - 2 = \frac{1}{2}(N-3) - \mu_2 \end{aligned} \tag{3.4}$$

by construction. In particular $N^* = N$ and (3.3) holds for the starred quantities, as well. By duality we may therefore skip the cases $\mu_2 > \mu_2^* = \frac{1}{2}(N-3) - \mu_2$, for now, and only consider cases N, μ_2 with

$$0 \leq \mu_2 \leq \frac{1}{4}(N-3). \tag{3.5}$$

Throughout we present stacked and face glued cases first, which derive from lower N. For $N \equiv 3 \pmod 4$, i.e., $N = 3, 7, 11$, we include the duals when $\mu_2 = \mu_2^* = \frac{1}{4}(N-3) = 0, 1, 2$. Enumerating the cases is delicate and remains a matter of taste, probably, at least as long as the counting problem for plane attractors remains unresolved. We choose a notation

$$N \cdot n^{m_n}(n-1)^{m_{n-1}} \dots 1^{m_1} - \ell \tag{3.6}$$

where n^{m_n} indicates a count m_n of n-gon faces in the 1-skeleton $G = \mathcal{C}^1$. An edge without face is assigned $n = 1$. Since multiple cases do arise, the number ℓ simply enumerates them, in somewhat arbitrary order. For $N < 21$ equilibria, n-gon faces with $n \geq 10$ do not arise. We can therefore safely omit exponents $m_n = 1$. The total face count is

$$\mu_2 = m_2 + \dots + m_n. \tag{3.7}$$

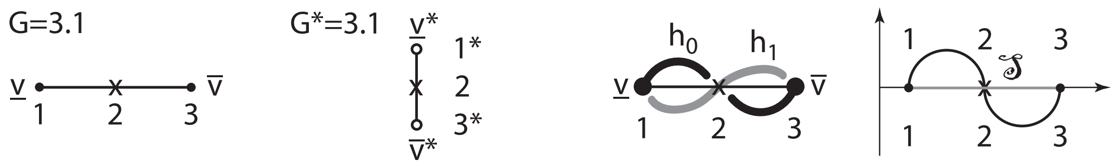

Figure 3.1: $N = 3$ equilibria. Left: 1-skeleton G and dual G^*. Right: paths h_0, h_1 and shooting curve $\mathcal{S}$.

Figure 3.2: $N = 5$ equilibria; straight line and dual Chafee-Infante 2-gon.

3.1 $N = 3$ equilibria

By (3.5), (3.3) we have $\mu_2 = 0$ and $\mu_0 = 2$, $\mu_1 = 1$. The straight line G with 2 end points $\underline{v}$, $\bar{v}$ is the only case. This case $G = 3.1$ is self dual. See figure 3.1.

3.2 $N = 5$ equilibria

Up to duality, we may still assume $\mu_2 = 0$. Hence $\mu_0 = 3$, $\mu_1 = 2$. Therefore the 1-skeleton G is the straight line, again, this time with 3 sinks. This case $G = 5.1^2$ can be obtained by stacking the case $G = 3.1$ onto itself; h_0, h_1, π remain trivial; see figure 3.2. Note however that the Chafee-Infante case $G = 5.2$ of a 2-gon attractor, as discussed in section 2.6, is dual to the straight line. Somewhat trivially, but alternatively, the 2-gon attractor can also be viewed as a face-gluing of the line attractor $N = 3$, on the left boundary, with another copy of itself, on the right boundary.

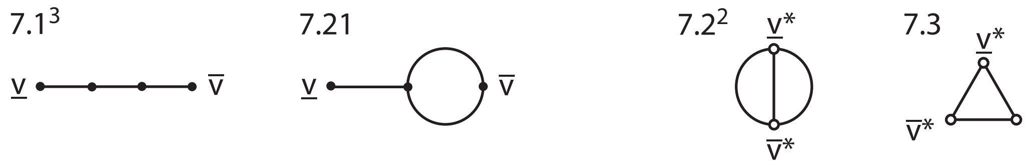

Figure 3.3: $N = 7$ equilibria. Left: $\mu_2 = 0, 1$. Right: duals $\mu_2 = 2, 1$.

3.3 $N = 7$ equilibria

Up to duality, we now have the one-dimensional subcase $\mu_2 = 0$ with $\mu_0 = 4$ sinks, and the case $\mu_2 = 1$ of a single face. The one-dimensional graph with $\mu_2 = 0$ faces cannot branch, because it must possess a Hamiltonian pair. Therefore we only obtain a line $G = 7.1^3$. For $\mu_2 = 1$ the single face is an n-gon, with $n = 2$ or 3, which adsorbs $2n + 1 = 5$ or 7 equilibria, respectively. The two cases $G = 7.21$ and $G = 7.3$ are mutually dual. All cases are stacked or glued. See figure 3.3 for the two 1-skeletons and their duals. The associated Sturm permutations are easily derived from sections 2.4 and 2.6.

3.4 $N = 9$ equilibria

This case has already been enumerated in [Fi94], even without the restriction of planarity; see figure 3.4 and [FiRo07b]. We have indicated the (filled) connection graphs $\mathcal{C}_f$, the Sturm permutations π_f, and the directions of heteroclinic orbits between equilibria of adjacent Morse index.
Originally these results had been obtained, somewhat mindlessly, by brute force computer assisted scanning of the permutations $\pi \in S_9$ for dissipative Morse meanders. In contrast, we now aim for an intelligible and more systematic derivation of all planar cases: the *Leitmotiv* is progress from scientific computing toward scientific understanding.
We follow the general duality approach outlined in (3.3)–(3.6). For $\mu_2 = 0$, dual to $\mu_2 = 3$, we again obtain the stacked line, only. For $\mu_2 = 1$, dual to $\mu_2 = 2$, we obtain a single n-gon with $n = 2, 3, 4$ edges and with 5,7,9 equilibria. For $n = 2$, the remaining two edges of G can be attached to the 2-gon in two stacking configurations. The omitted cases are just substitution equivalent or orientation isomorphic copies. A similar remark holds for the substitution equivalent or orientation isomorphic edge-face pairs, when $n = 3$ and the free edge is attached to an extremum $\underline{v}'$ or $\bar{v}'$ of the 3-gon. We cannot attach the edge to the third sink v on the face boundary, different from $\underline{v}'$, $\bar{v}'$: this would preclude Hamiltonian paths from $\underline{v}$ to $\bar{v}$ to exist. The face glued final two cases of an n-gon, $n = 4$, without

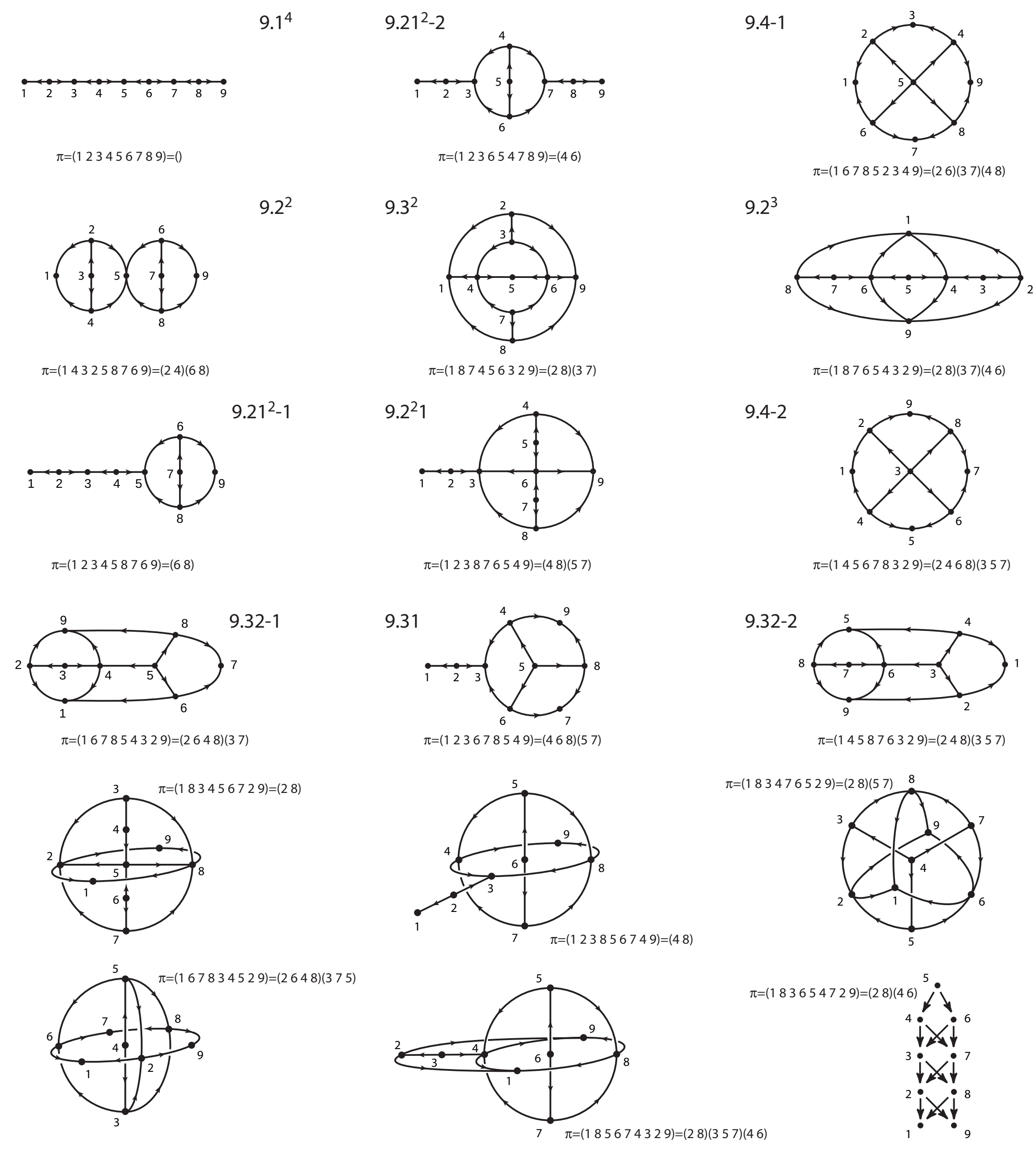

Figure 3.4: All 18 connection graphs $\mathcal{C}_f$ of Sturm attractors $\mathcal{A}_f$ with $N = 9$ equilibria, up to trivial substitution equivalence. Equilibria are numbered such that $h_0 = id$; in particular $\underline{v} = 1$ and $\bar{v} = 9$. See [Fi94] and [FiRo07b]. For case numbers of planar cases see figure 3.5
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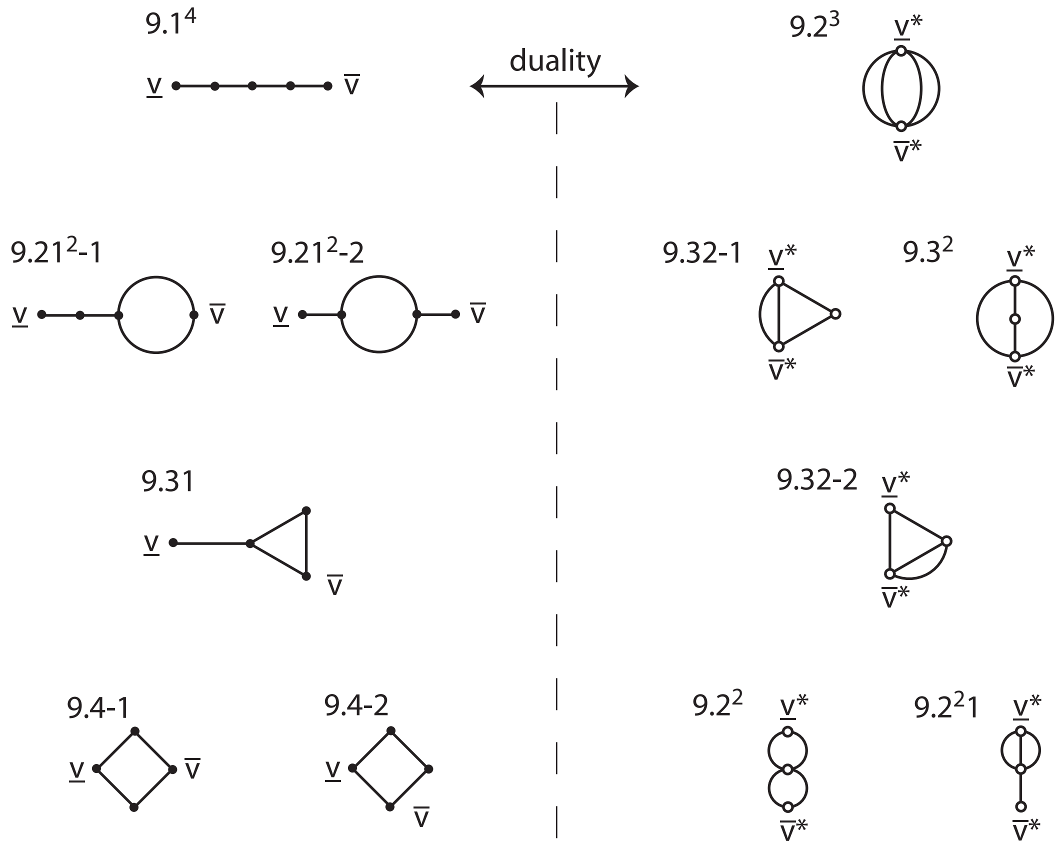

Figure 3.5: The Sturm 1-skeletons with $N = 9$ equilibria. Left: $\mu = 0, 1$ stacked and glued cases. Right: $\mu_2 = 3, 2$ duals of left side. For enumeration of cases see (3.6).

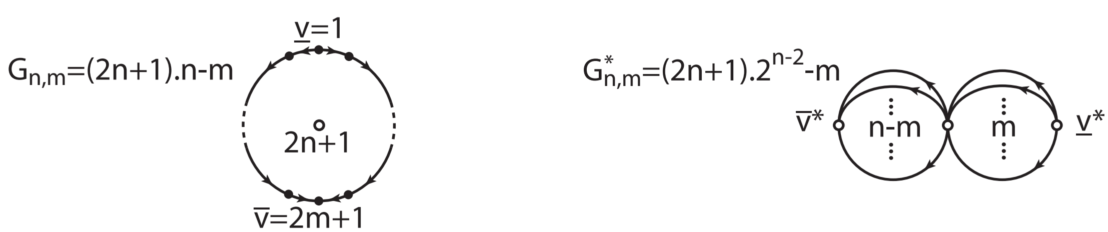

Figure 3.6: Isomorphic n-gon attractors with $N = 2n + 1$ equilibria. Left: non-isomorphic orientations on $G = \mathcal{C}^1_{n,m}$. Right: non-isomorphic duals G^*.

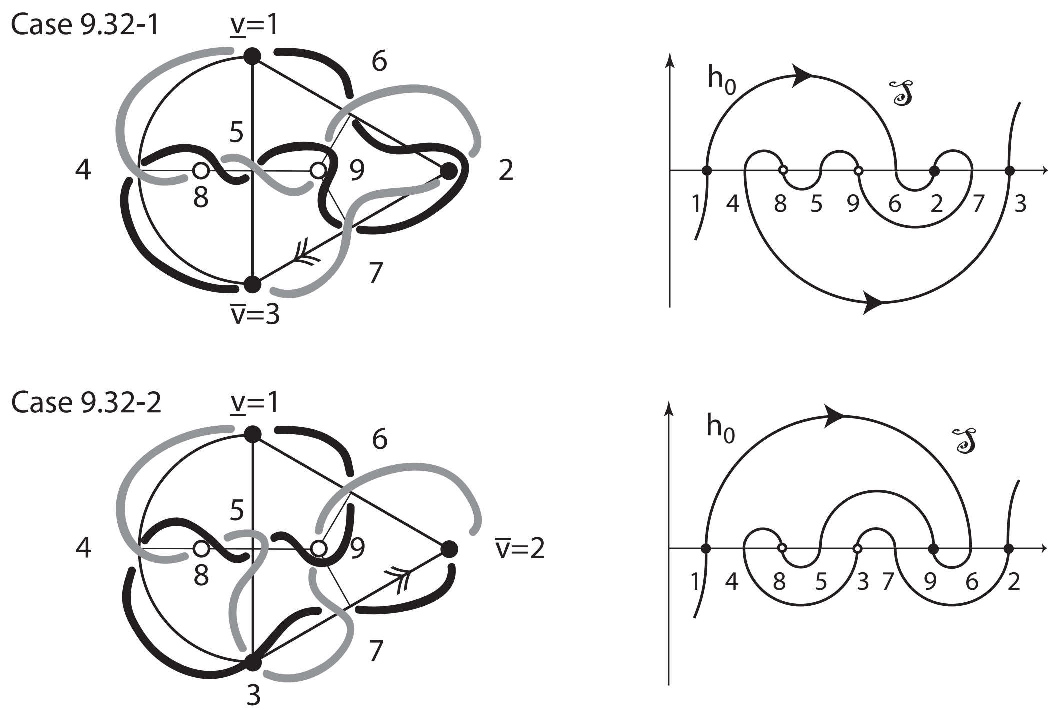

Figure 3.7: Isomorphic, but neither orientation isomorphic nor substitution equivalent, connection graphs $9.32 - \ell$ with $\ell = 1, 2$. Left: disentangled boundary ZS-Hamiltonian paths h_0 (black) and h_1 (gray). Right: shooting curves.

attached edges, are illustrated in figure 3.5 together with their stacked and glued duals. Clearly $\mu_2 = 0, 1$ faces are dual to $\mu_2^* = 3, 2$ faces, respectively.

Even though all examples can be obtained by gluing and stacking, there are two pairs of isomorphic attractor 1-skeletons G here, which are neither substitution equivalent nor orientation isomorphic. For the 4-gon cases 9.4-1 and 9.4-2 see the second row of figure 1.2. See also figure 2.5, $n = 4$, $m = 2, 3$ for (h_0, h_1) pairs in $\mathcal{A}_{4,3}$ and $\mathcal{A}_{4,2}$. For the associated Sturm permutations see (2.24) and figure 3.4. These cases are clearly distinguished by their non-isomorphic duals G^*. The general case $\mathcal{A}_{n,m}$, $G_{n,m} = \mathcal{C}^1_{n,m}$ of an n-gon with $\underline{v} = 1$ and $\bar{v} = 2m + 1$, and its dual $G^*_{n,m}$ is sketched in figure 3.6. Note how $G_{n,m}$ is face glued from two lines: one with $m + 1$ sinks, on the right, and the other with $(n - m) + 1$ sinks, on the left. Similarly, $G^*_{n,m}$ consists of two stacked parts. We recall from section 2.6 that $G_{n,m}$ and $G_{n,m'}$ are neither orientation isomorphic nor substitution equivalent, for $m \neq m'$, unless $m + m' = n$.

The other isomorphic, but neither substitution equivalent nor orientation isomorphic, pair involves one 3-gon with one attached 2-gon; see the duals 9.32-1 and 9.32-2 in figure 3.5. Again, the orientation discrepancy is caused by different positioning of the extrema $\underline{v}, \bar{v}$. In figure 3.7 we sketch the orientations, the unique resulting boundary (ZS)-Hamiltonian pairs, and the associated shooting curves. The Sturm permutations $\pi_1 = (2\ 8\ 4\ 6)(3\ 7)$ in the top row, and $\pi_2 = (2\ 6\ 8)(3\ 5\ 7)$ in the bottom row, are trivially substitution equivalent to their respective counterparts in figure 3.4. Moreover π_1 and π_2 produce isomorphic Sturm attractors. But π_1 and π_2 are not even conjugate. This has been noted in [Fi94] and is resolved here, in terms of different orientations on the 1-skeleton $G = \mathcal{C}^1$. See [Wo02b] for an interesting geometric interpretation in terms of fast unstable manifolds.

3.5 $N = 11$ equilibria

This time we can have $\mu_2 = 0, 1$, or 2 faces, up to duality. The line case $\mu_2 = 0$, and the obvious configurations of the single n-gon face, $n = 2, 3, 4, 5$ in the case $\mu_2 = 1$ are illustrated by their 1-skeletons in figure 3.8, along with their duals. See also section 2.6. Substitution equivalent and orientation isomorphic copies are eliminated.

By direct inspection, all connection graphs with $\mu_2 \neq 2$ faces are stacked or glued. We have

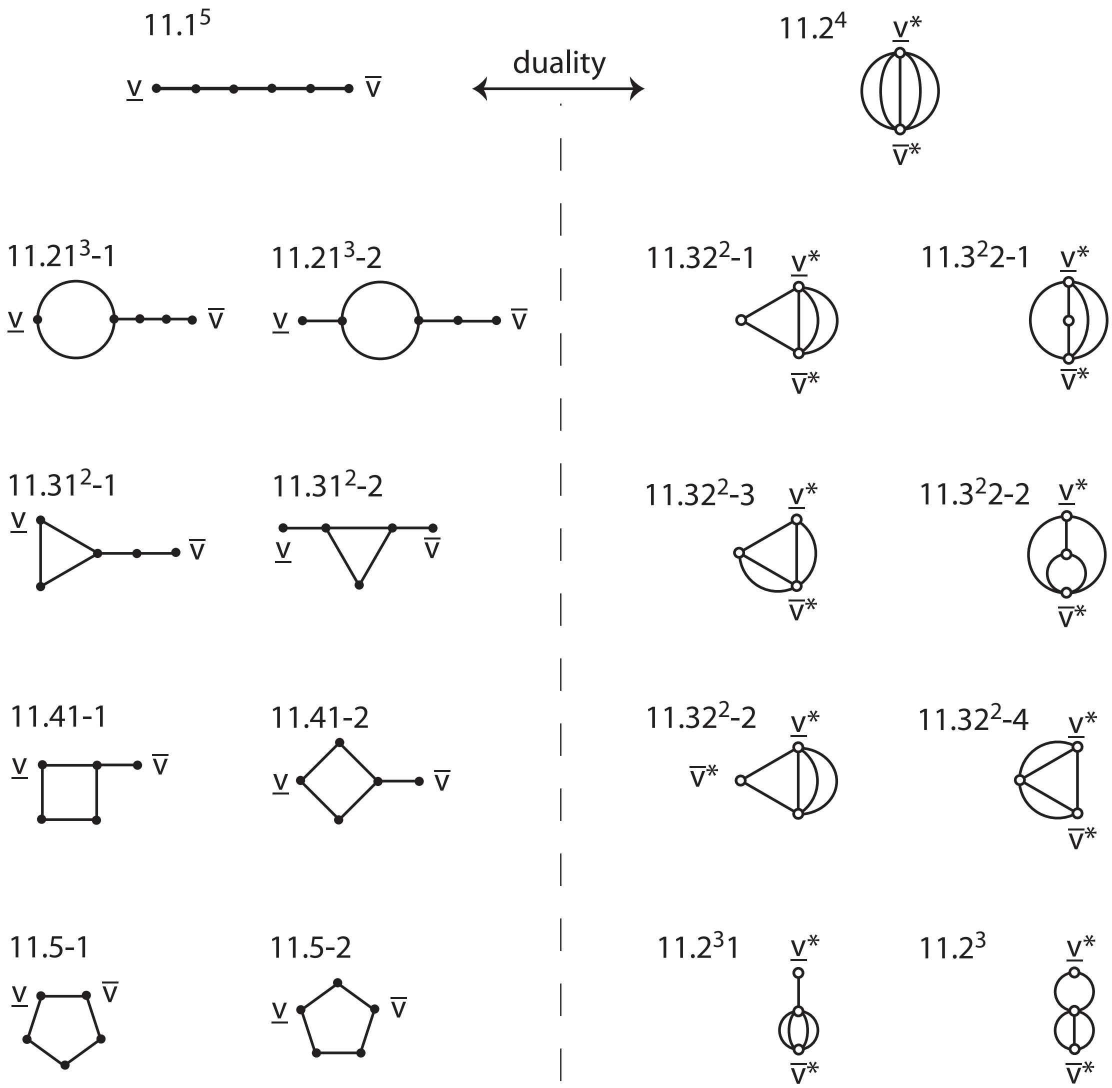

Figure 3.8: The Sturm 1-skeletons with $N = 11$ equilibria and $\mu_2 \neq 2$ faces, ordered by single n-gons, $n = 2, 3, 4, 5$, and paired with their duals. For Sturm permutations and enumeration of cases see (3.6) and table 3.1.

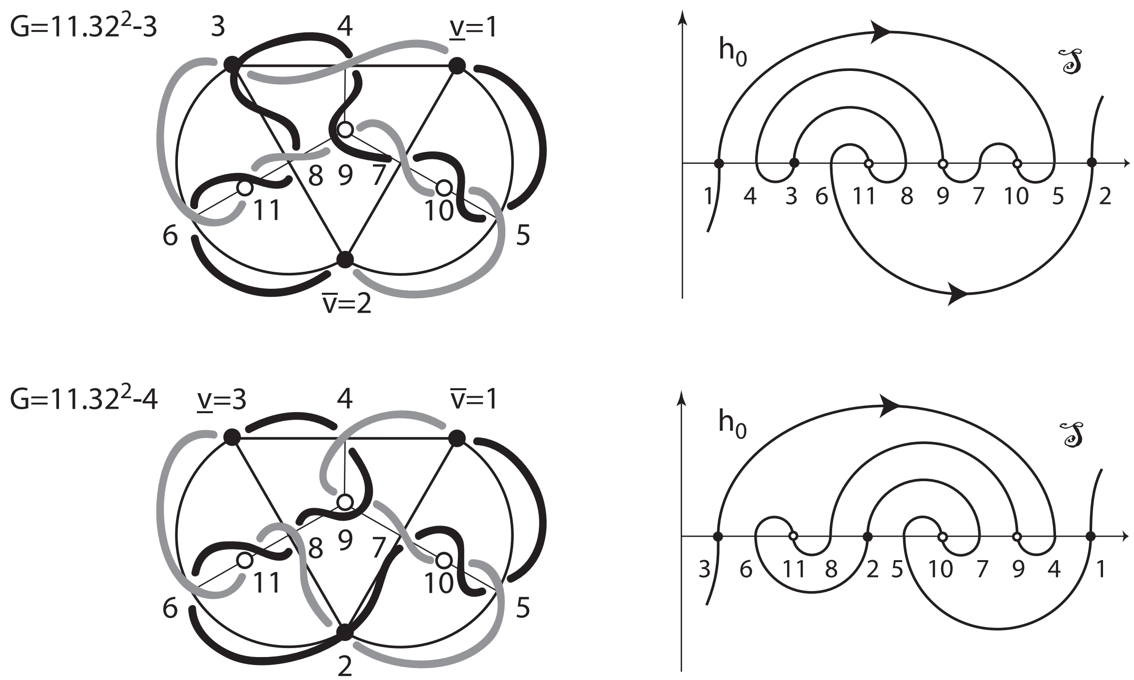

Figure 3.9: Isomorphic, but neither orientation isomorphic nor substitution equivalent, "snoopy" attractors $11.32^2 - \ell$ with $\ell = 3, 4$. Left: disentangled boundary ZS-Hamiltonian paths h_0 (black) and h_1 (gray). Right: shooting curves.

four pairs of C^0 orbit equivalent attractors, among these, which carry different orientations:

$$\begin{aligned}
&\{11.32^2-1, \quad 11.32^2-2\},\\
&\{11.32^2-3, \quad 11.32^2-4\},\\
&\{11.41-1, \quad 11.41-2\},\\
&\{11.5-1, \quad 11.5-2\}.
\end{aligned} \tag{3.8}$$

The first and third pairs are similar to the simpler pairs {9.32-1, 9.32-2} and {9.4-1, 9.4-2} discussed in subsection 3.4, due to a face gluing and a line stacking, respectively. The second pair is illustrated in figure 3.9. The last pair, the 5-gon, has been discussed in section 2.6 in complete generality. See also the third row in figure 1.2.

We now turn to the remaining case of connection graphs $\mathcal{C}$ with $N = 11$ vertices and $\mu_2 = 2$ faces. Again we proceed graphically by side numbers $n_1 \geq n_2$ of the two n-gons. Duals also possess $\mu_2^* = 2$ faces. We therefore include duals, for each case, until we have exhausted all possibilities. All cases are stacked and glued versions of constituents which we have encountered already – with one exception. See figure 3.10. Case $G = 11.3^2 - 3 = G^*$ is the first self-dual case after the trivial line $G = 3.1$. By absence of a decomposing cut-vertex, it is neither stacked nor glued.

Isomorphic Sturm attractors with deviating orientations of their 1-skeletons arise in the following cases:

$$\begin{aligned}
&\{11.321-2, \quad 11.321-3\},\\
&\{11.3^2-1, \quad 11.3^2-2, \quad 11.3^2-3\},\\
&\{11.42-1, \quad 11.42-2, \quad 11.42-3, \quad 11.42-4\},\\
&\{11.43-1, \quad 11.43-2\}.
\end{aligned} \tag{3.9}$$

The last pair originates from the simpler pair $\{9.32-1,\ 9.32-2\}$ illustrated in figure 3.7. We illustrate the triplet cases of isomorphic, but neither substitution equivalent nor orientation isomorphic, Sturm attractors $11.3^2 - \ell$ in figure 3.11, which also includes the self-dual case $11.3^2 - 3$.

In tables 3.1, 3.2, we summarize all 37 Sturm permutations $\pi = \pi_f$ with $N = 11$ hyperbolic equilibria which give rise to plane attractors $\dim \mathcal{A} \leq 2$. For any listed permutation π we omit substitution equivalent and orientation isomorphic variants. The upper permutation

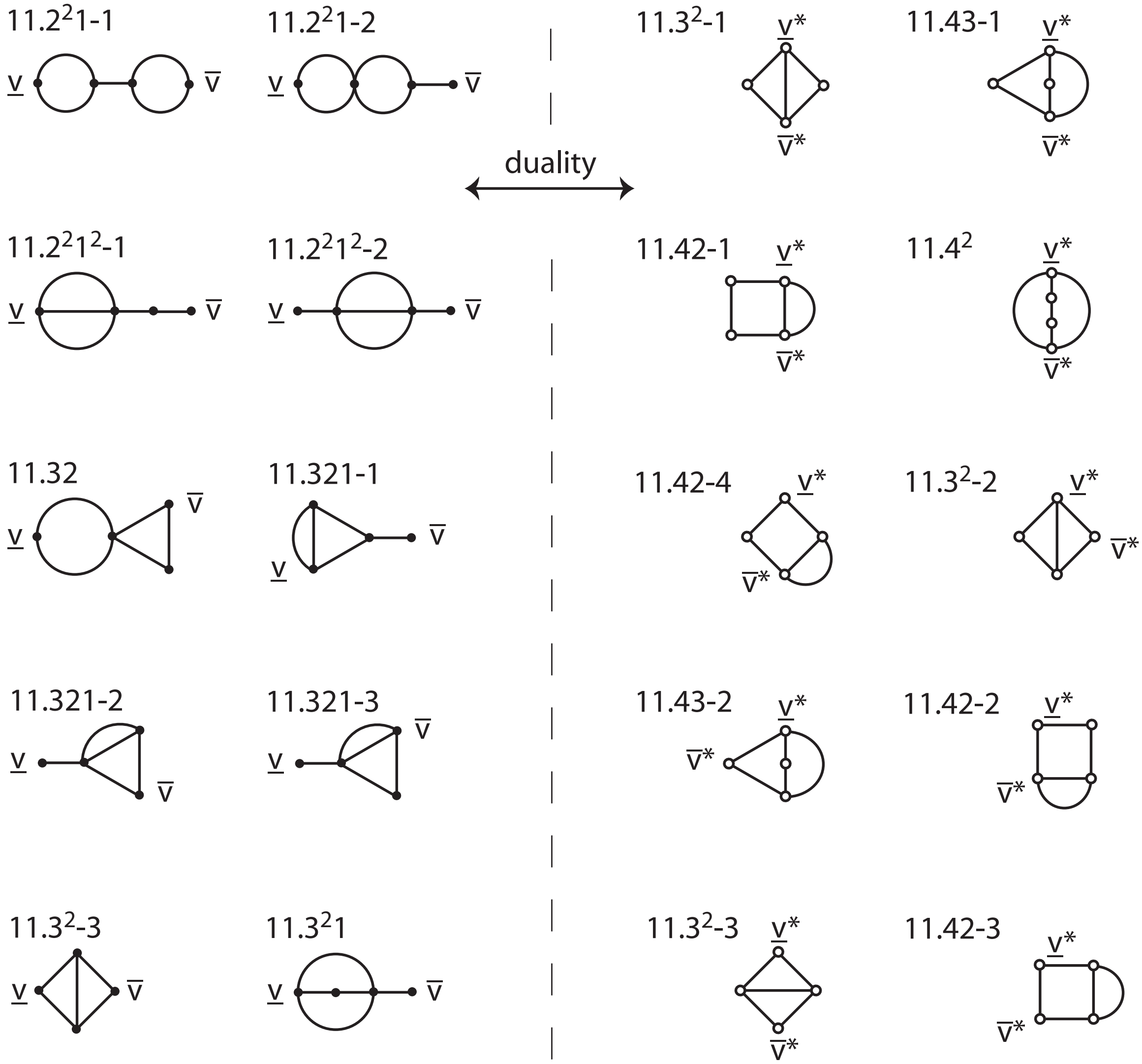

Figure 3.10: The Sturm 1-skeletons with $N = 11$ equilibria and $\mu_2 = 2$ faces, paired with their duals. For Sturm permutations and notation of cases see (3.6) and table 3.2

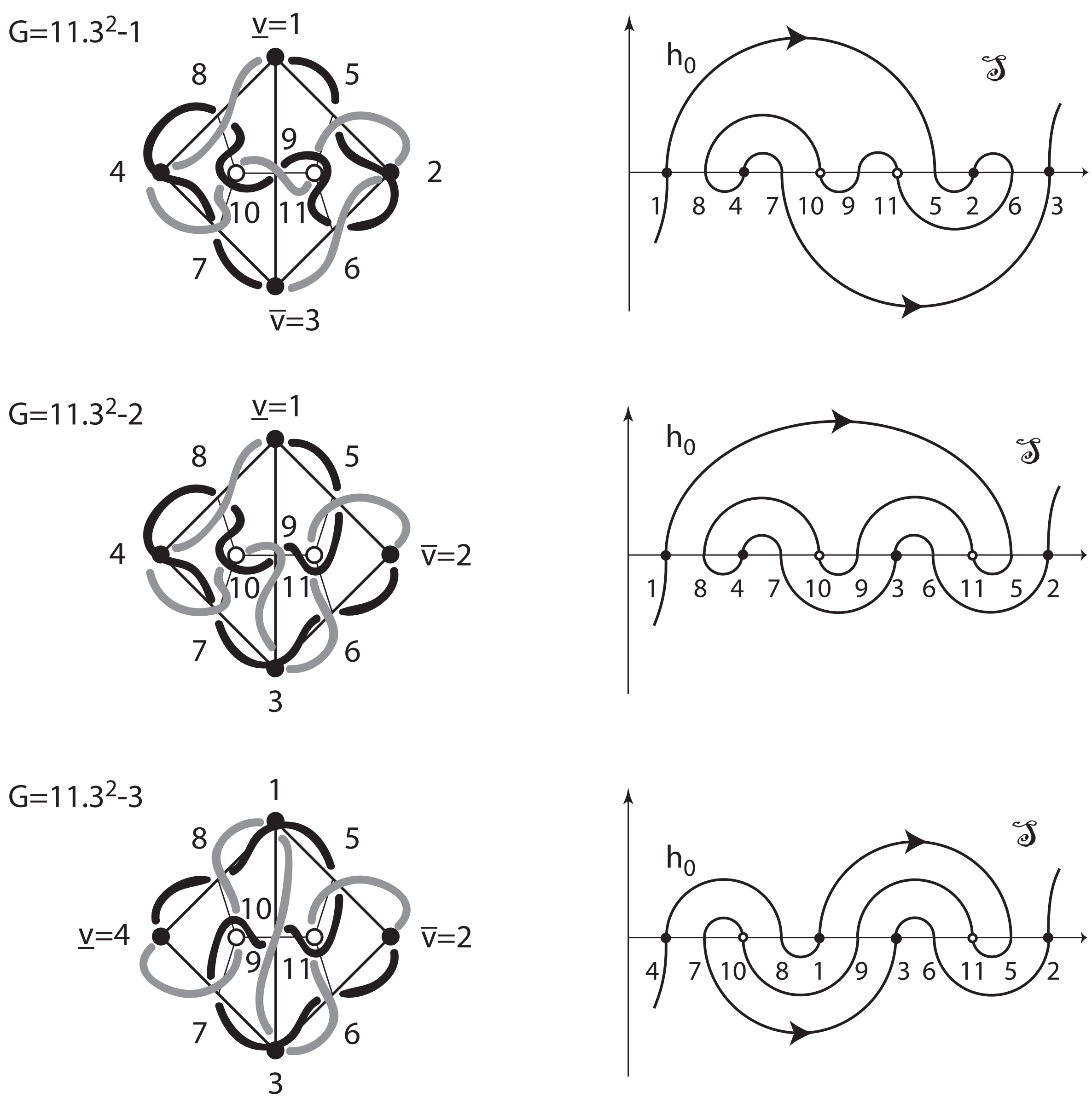

Figure 3.11: Isomorphic, but neither orientation isomorphic nor substitution equivalent, attractor triplet $11.32^2 - \ell$ with $\ell = 1, 2, 3$. Left: disentangled boundary ZS-Hamiltonian paths h_0 (black) and h_1 (gray). Right: shooting curves. Bottom row: self-duality.

μ_2	11.case	11.dual	permutation π	in cycles
0	1^5	2^4	2 3 4 5 6 7 8 9 10	( )
1	21^3-1	32^2-1	4 3 2 5 6 7 8 9 10	(2 4)
	21^3-2	3^22-1	2 3 6 5 4 7 8 9 10	(4 6)
	31^2-1	32^2-3	4 5 6 3 2 7 8 9 10	(2 4 6) (3 5)
	31^2-2	3^22-2	2 3 6 7 8 5 4 9 10	(4 6 8) (5 7)
	$41-1$	32^2-2	4 5 6 7 8 3 2 9 10	(2 4 6 8) (3 5 7)
	$41-2$	32^2-4	6 7 8 5 2 3 4 9 10	(2 6) (3 7) (4 8)
	$5-1$	2^31	4 5 6 7 8 9 10 3 2	(2 4 6 8 10) (3 5 7 9)
	$5-2$	2^3	6 7 8 9 10 5 2 3 4	(2 6 10 4 8) (3 7 5 9)
3	2^3	$5-2$	4 3 2 5 10 9 8 7 6	(2 4) (6 10) (7 9)
	2^31	$5-1$	2 3 10 9 8 7 6 5 4	(4 10) (5 9) (6 8)
	32^2-1	21^3-1	8 9 10 7 6 5 4 3 2	(2 8 4 10) (3 9) (5 7)
	32^2-2	$41-1$	4 5 10 9 8 7 6 3 2	(2 4 10) (3 5 9) (6 8)
	32^2-3	31^2-1	6 7 10 9 8 5 4 3 2	(2 6 8 4 10) (3 7 5 9)
	32^2-4	$41-2$	6 5 4 7 10 9 8 3 2	(2 6 10) (3 5 7 9)
	3^22-1	21^3-2	10 9 6 7 8 5 4 3 2	(2 10) (3 9)(4 6 8) (5 7)
	3^22-2	31^2-2	10 9 4 5 8 7 6 3 2	(2 10) (3 9) (6 8)
4	2^4	1^5	10 9 8 7 6 5 4 3 2	(2 10) (3 9) (4 8) (5 7)

Table 3.1: Duality and planar Sturm permutations with $N = 11$ equilibria. Enumeration of all cases with $\mu_2 \neq 2$ faces, up to trivial substitutions and orientation isomorphisms. Dissipative ends $\pi(1) = 1$, $\pi(11) = 11$ are omitted. See also figure 3.8.

entries of table 3.1 are easily generated, even by hand, from the n-gon formula (2.24), and the stacking principle of section 2.3. The lower dual entries with $\mu_2 = 3$ and 4 faces then follow from (2.21).

3.6 Self-duality

All permutation entries of table 3.2, $\mu_2 = 2$, but one, originate from simpler constituents by stacking and duality. The one exception is the self dual case $11.3^2 - 3$. It is easy to derive this unique self dual case with $N = 11$ equilibria directly. By (2.21), self-duality implies $\pi = \hat{\kappa}\pi^{-1}$. In other words,

(3.10) $$\pi^2 = \hat{\kappa},$$

11.case	11.dual	permutation π	in cycles
$2^2 1 - 1$	$3^2 - 1$	4 3 2 5 6 7 10 9 8	(2 4) (8 10)
$2^2 1 - 2$	$43 - 1$	4 3 2 5 8 7 6 9 10	(2 4) (6 8)
$2^2 1^2 - 1$	$42 - 1$	6 5 4 3 2 7 8 9 10	(2 6) (3 5)
$2^2 1^2 - 2$	4^2	2 3 8 7 6 5 4 9 10	(4 8) (5 7)
32	$42 - 4$	4 3 2 5 8 9 10 7 6	(2 4) (6 8 10) (7 9)
$321 - 1$	$3^2 - 2$	8 7 4 3 2 5 6 9 10	(2 8 6) (3 7 5)
$321 - 2$	$43 - 2$	2 3 10 9 6 5 4 7 8	(4 10 8) (5 9 7)
$321 - 3$	$42 - 2$	2 3 8 9 10 7 6 5 4	(4 8 6 10) (5 9)
$3^2 1$	$42 - 3$	8 7 4 5 6 3 2 9 10	(2 8) (3 7)
$3^2 - 3$	self-dual	8 7 2 3 6 9 10 5 4	(2 8 10 4) (3 7 9 5)
$3^2 - 1$	$2^2 1 - 1$	8 9 10 7 6 5 2 3 4	(2 8) (3 9) (4 10) (5 7)
$3^2 - 2$	$321 - 1$	6 7 8 5 4 9 10 3 2	(2 6 4 8 10) (3 7 9)
$42 - 1$	$2^2 1^2 - 1$	6 7 8 9 10 5 4 3 2	(2 6 10) (3 7 5 9) (4 8)
$42 - 2$	$321 - 3$	10 9 2 3 4 5 8 7 6	(2 10 6 4) (3 9 7 5)
$42 - 3$	$3^2 1$	4 5 8 7 6 9 10 3 2	(2 4 8 10) (3 5 7 9)
$42 - 4$	32	8 9 10 7 2 3 6 5 4	(2 8 6) (3 9 5 7) (4 10)
$43 - 1$	$2^2 1 - 2$	8 9 10 7 4 5 6 3 2	(2 8 6 4 10) (3 9) (7 5)
$43 - 2$	321-2	10 9 4 5 6 3 2 7 8	(2 10 8) (3 9 7)
4^2	$2^2 1^2 - 2$	10 9 4 5 6 7 8 3 2	(2 10) (3 9)

Table 3.2: Duality and planar Sturm permutations with $N = 11$ equilibria. Enumeration of all cases with $\mu_2 = 2$ faces, up to trivial substitutions and orientation isomorphisms. Dissipative ends $\pi(1) = 1$, $\pi(11) = 11$ are omitted. See also figure 3.10.

and hence π is a square root of the involution $\hat{\kappa} = (2\ 10)\ (3\ 9)\ (4\ 8)\ (5\ 7)$. Therefore π must consist of 4-cycles. Since π preserves parity mod 2, like any dissipative meander, the 4-cycles must permute the sets $\{2, 4, 8, 10\}$ and $\{3, 5, 7, 9\}$, separately. Since (3.10) holds for the entire substitution orbit π^{-1}, $\kappa\pi\kappa$, $\kappa\pi^{-1}\kappa$, once it holds for π itself, only the two candidates $\pi = (2\ 8\ 10\ 4)\ (3\ 7\ 9\ 5)$ and $(2\ 8\ 10\ 4)\ (3\ 5\ 9\ 7)$ remain, without loss of generality. Since (2 8 10 4) (3 5 9 7) is not a meander permutation, this proves

$$\pi = (2\ 8\ 10\ 4)\ (3\ 7\ 9\ 5), \tag{3.11}$$

to be the only self-dual case with $N = 11$ equilibria, up to trivial substitutions.

Motivated by the pitchforkable Chafee-Infante nonlinearity $f = \lambda u(1 - u^2)$, as discussed in [He85], [CoSm83], the self-dual case $11.3^2 - 3$ has already been considered in [Ro91] as an example of a non-pitchforkable Sturm attractor: *pitchforkable* attractors can be simplified by a pitchfork bifurcation which reduces the number of equilibria by 2. In terms of the Sturm permutation π, they feature three adjacent entries:

$$\begin{array}{ll} \pi = & (\ldots m-1 \quad m \quad m+1 \ldots) \quad \text{or} \\ \pi = & (\ldots m+1 \quad m \quad m-1 \ldots). \end{array} \tag{3.12}$$

By tables 3.1, 3.2 and their simpler ancestors with $N = 3, 5, 7$ and 9 equilibria, the self-dual attractor $11.3^2 - 3$ is in fact the first and lowest-dimensional non-pitchforkable example with up to 11 equilibria.

4 Specifics: the planar Platonic graphs

In sections 4.1–4.5 we design Sturm attractors with the five planar Platonic graphs G as 1-skeletons $\mathcal{C}_f^1$ of their connection graphs. Throughout we eliminate cases with substitution equivalent attractors or orientation isomorphic 1-skeletons G.

Polarity is a primary feature of Sturm attractors and, more generally, of any Morse system. This is caused by the two possible signs of any eigenfunction for any simple real eigenvalue and, consequently, by the two asymptotic directions of any trajectory which converges to any hyperbolic equilibrium – be it forward or backward in time, at a slower or faster exponential rate. See for example [BrFi86].

Such polarity is not easily accomodated by Platonic polyhedra. In the following sections we therefore present our attempts at designing planar Sturm attractors $\mathcal{A}_f$ such that the 1-skeletons $\mathcal{C}_f^1$ of their connection graphs coincide with the five Platonic graphs. Each of these planar graphs possesses only one plane embedding G, up to isomorphism, by the symmetries of the Platonic solids.